CBE 30356 Transport II
Problem Set 10
Due via Gradescope, 11:55 PM 4/13/23

1). During the pandemic it became clear that a key way (probably *the* key way) the virus is transmitted is via aerosols. While large droplets fall rapidly to the ground, smaller drops become aerosols that float about, waiting to be inhaled. The amount of virus a drop contains is proportional to its volume – so the transition of a large drop into a small one via evaporation is critical. In this problem I want you to answer a simple question: What is the largest drop of water falling from an altitude of 1.6 meters (e.g., about where the average mouth is) that evaporates before hitting the ground, and how does this compare to the roughly 25$\mu$m diameter drops I emit when I say "puppy" or "polymer"? For this calculation, take the air to be at 22°C at 40% relative humidity.

While the Reynolds number of the falling droplets is small, it's not zero. Many correlations for mass, momentum, and energy are available, but for your calculations use the following:

$$\frac{F}{6\pi\mu Ua} \approx 1 + \frac{1}{6}\mathrm{Re}^{1/2} + \frac{1}{60}\mathrm{Re}$$

$$Nu \approx 2 + 0.6\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

$$Sh \approx 2 + 0.6\,\mathrm{Re}^{1/2}\,Sc^{1/3}$$

Use the Buck Equation for the vapor pressure of the water. Note that a saliva droplet would behave a little differently because of the stuff dissolved in it, but it is still 99% water. As a result, the droplet nuclei left behind is small enough to float about in the air, and the calculation for a clean drop of water (e.g., what I'm asking you to do) gets it about right.

a. Ignoring convection (e.g., if the Reynolds number is zero in the correlations above), you can actually solve this analytically once you've got the equilibrium concentration at the surface (which as you recall is not a function of drop size). In this limit the sedimentation velocity of the drop is proportional to the radius squared, which in turn is linear in time, so it's easy to get h as a function of t (and thus r). If you take h = 0 as the point where R = 0, then you can determine the radius of our evaporating drop at any height. Do this to determine the value of $R_0$ where h = 1.6m.

b. The Reynolds number isn't zero, however, so your complete solution will have to be numerical. The simplest way to approach it is to pick an initial droplet size, calculate the velocity via root finding on the drag equation, and then solve the coupled mass and energy transport problem via the Ranz-Marshall equations above. This should give you an evaporation rate as a function of drop radius, which you can then use to get a change in height as well as radius as a function of time. If you then plot up h as a function of R you would get a master curve for all drops, and taking h = 0 as the point where R = 0 you can determine the radius at a height of 1.6m, the same as was done for your analytic solution ignoring the Reynolds number dependence.