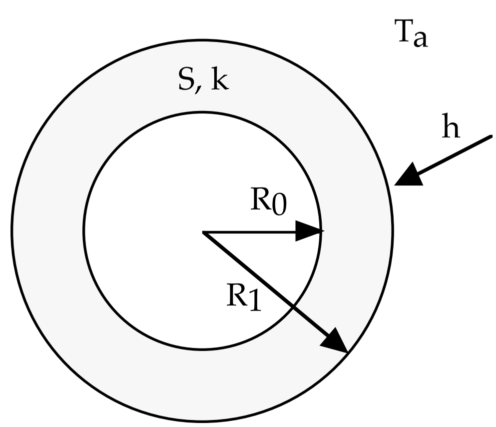
CBE 30356 Transport II
Problem Set 2
Due via Gradescope, 9 PM 2/2/23

1). A hot wire anemometer is a method for measuring air velocity by determining the rate of heat transfer and temperature from a heated wire. The idea is that because of electrical dissipation in a *very* thin wire it gets hot – and the temperature will depend on the air flow rate cooling it down. The temperature is measured from the resistance of the wire. Usually you measure changes in the voltage drop in what is called a Wheatstone Bridge circuit, however here we will look at a simpler problem. Suppose your wire is L = 1 cm long and D = 1mm in diameter. It has a resistance Ω_0 of 1 ohm at 20°C and a temperature coefficient λ of 2.45×10^{-3} 1/°C (assume linearity). You operate the probe at a fixed current of I = 0.5 Amps. Calculate the following:

a. If we have a wind speed of U = 1m/s, what is the temperature of the probe? Neglect internal resistance (e.g., low Biot number as is appropriate for this problem) and use the Whittaker correlation to get the heat transfer coefficient. Ignore variation of fluid properties with temperature (just keep the temperature dependence of the electrical resistance!). Note that the power dissipation goes up a little bit with temperature too.

b. As the temperature and resistance change, so does the voltage drop V. Using a semilog scale, plot the measured voltage across the probe (the current is kept constant) as a function of wind speed over the range 1 m/s to 100 m/s.

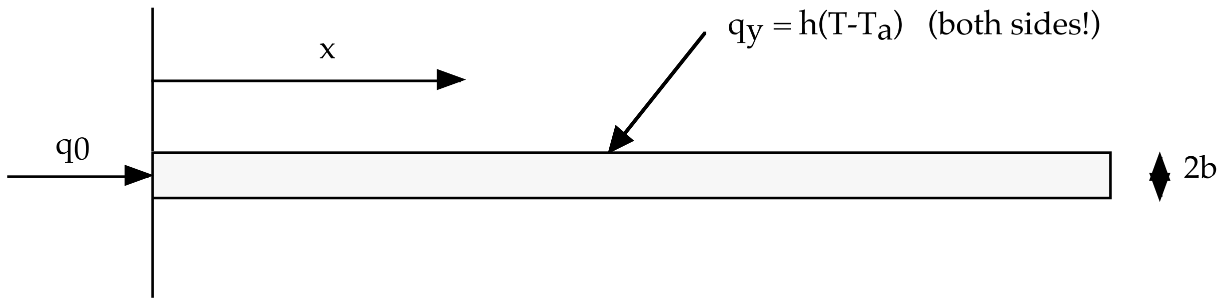
2). In this problem we will look at an annular energy source as depicted below. A radioactive source is confined to an annulus between an inner radius R_0 and an outer radius R_1, and releases heat at a rate S per unit volume. The material has a thermal conductivity k, and the outside is cooled with a heat transfer coefficient h relative to an atmospheric temperature T_a. Because there is nothing inside the radius R_0, the radial heat flux at that point is zero.



a. Using an energy balance, calculate the temperature T_1 at the outer surface R_1.

b. Now solve for the temperature T_0 at the inner edge of the radioactive source R_0. This problem is much easier if you render radius dimensionless with the outer radius!

3). A cooling fin of width 2b is cooling a reactor as depicted below. The fin is sufficiently thin that you can ignore temperature variations across the width, but of course it changes along the length due to finite thermal conductivity k. If the heat transfer coefficient between the fin and the surrounding air (temperature T_a) is h (both sides), and the heat flux at the base of the fin is q_0, we can calculate the temperature at the base.

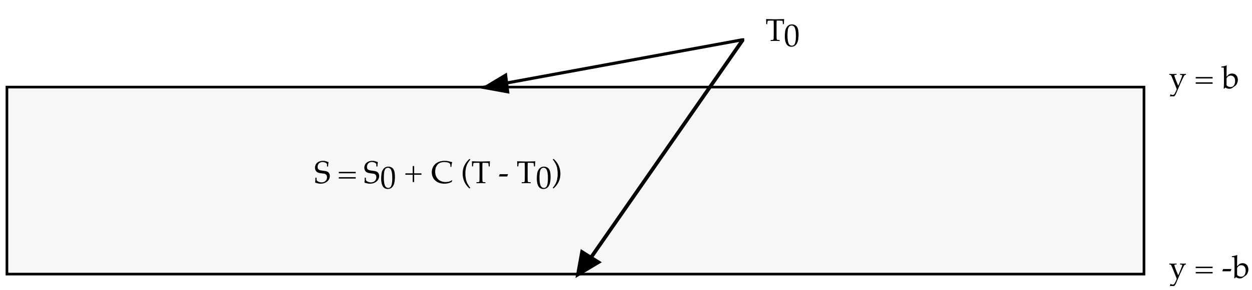


a. Write down the equations and boundary conditions for the temperature of the fin (averaged across its thin direction).

b. Render the equations dimensionless to determine the temperature scale and the characteristic length scale.

c. Assuming that the fin is "long" relative to this length scale, solve the problem and determine the temperature at the base.

4). A reaction is occurring in a slab of width 2b. It's exothermic, and is producing a lot of heat which is dissipated by cooling the walls at $y = \pm b$ to a temperature T_0. The problem is that the reaction rate increases with temperature, and this creates the possibility of thermal runaway in the center! Your job is to figure out what the temperature at the centerline is under these conditions.



a. While the temperature dependence is often complex, here we shall use the simple linear relationship $S = S_0 + C\,(T - T_0)$. If the thermal conductivity is a constant k, write down the differential equation and boundary conditions governing the temperature at steady-state.

b. Render the equation and boundary conditions dimensionless, determine the characteristic temperature rise, and show that the problem depends only on a single dimensionless ratio.

c. Now for the fun part: Solve for the temperature distribution as a function of y^* and plot it up for a few values of the dimensionless ratio you got in part b. For sufficiently large values of C there is no solution (and your reactor just melted down). What is this critical value?

Hint: You should obtain an inhomogeneous linear second order ODE for T^*. Remember that you most easily solve this sort of equation by finding the *particular solution* which removes the inhomogeniety. In this case it is just a constant! The leftover bit after subtracting the particular solution is a *homogeneous* second order equation which should be *very* familiar. Apply your boundary conditions to the sum of the particular and homogenous solutions and you are done!