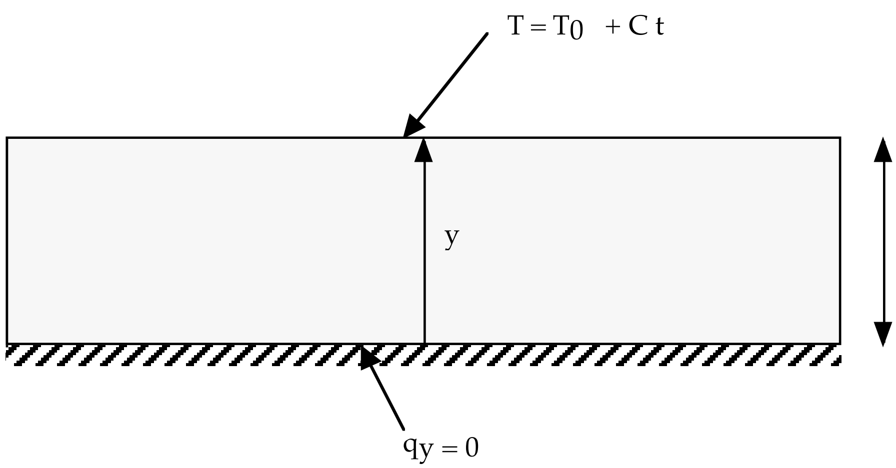
CBE 30356 Transport II
Problem Set 3
Due via Gradescope, 11:55 PM 2/9/23

1). Consider the slab of width d depicted below. Initially the entire slab is at a temperature T_0. The bottom at y = 0 is insulated ($q_y = 0$) and for $t > 0$ the upper wall at y = b is maintained at a temperature that increases in time, e.g., $T = T_0 + Ct$.

a. Write down the differential equation and boundary conditions that govern this problem, and render them dimensionless.

b. Calculate the asymptotic solution at long times and determine the temperature of the lower (insulated) plate. Note that because of this boundary condition it has to be a function of temperature too!

c. Calculate the heat flux through the upper wall, and using that and the answer to b determine the long-time heat transfer coefficient for this problem.



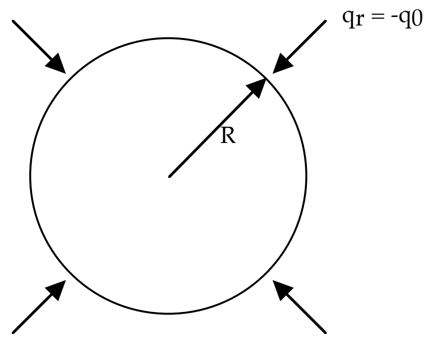
2). While the asymptotic solution is the most important, the transient solution is also useful! Using separation of variables, solve the Sturm-Liouville problem for the slab and boundary conditions above and determine the lead eigenvalue (e.g., how long do we need to wait for the heat transfer coefficient you calculated to become valid).

3). Consider the sphere of radius R depicted below. Initially it is at a temperature T_0, but we heat it uniformly at the surface with some value $q_r = -q_0$ (negative so that the temperature is going up rather than down!).

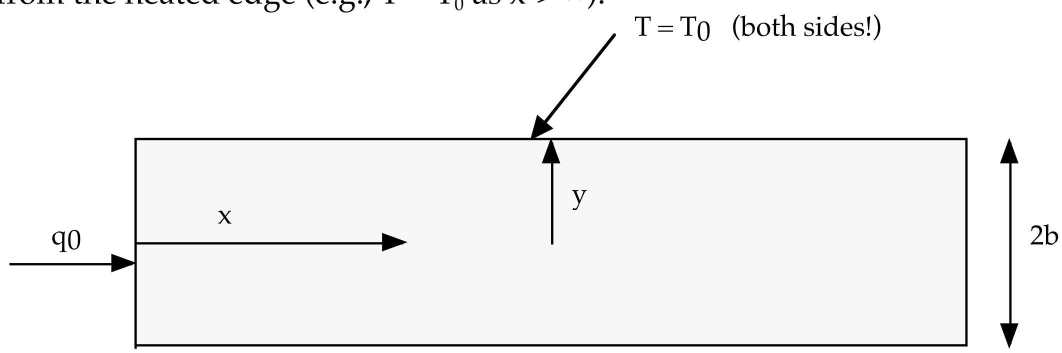
a. Write down the differential equation and boundary conditions that govern this problem and render them dimensionless.

b. Solve for the asymptotic solution at long times (which will be an increasing function of time!). Determine the temperature at the surface.

c. Calculate the average temperature of the sphere, and use that and the answer to b to determine the heat transfer coefficient for this problem.



4. We've been using Sturm-Liouville theory and separation of variables to look at parabolic PDE's such as the heat equation, however they can be used for elliptic PDE's as well (e.g., steady heat transfer in two dimensions!). Consider the slab depicted below. The walls at y ± b are maintained at a temperature T_0 at all time, and the end of the slab at x = 0 is heated with a constant heat flux $q_x = q_0$. The slab is long enough in the x direction that the temperature in the interior just approaches that of the wall far from the heated edge (e.g., $T = T_0$ as $x \to \infty$).



a. Write down the differential equation governing the problem and render it dimensionless. (Hint: it's at steady-state, and there's only one length scale…)

b. The variable x plays the same role as t in the transient problems we've been studying (although with a second derivative rather than a first!). Using this, show that it admits a separation of variables solution and write down the solution to the x part (decaying as $x \to \infty$).

c. Set up the Sturm-Liouville problem in the y direction and solve it, obtaining the eigenfunctions and eigenvalues.

d. Solve for the coefficients to get the complete solution.

e. Plot up the dimensionless temperature distribution in the y-direction for a few values of x, and plot the centerline temperature (y = 0) as a function of x.