CBE 30356 Transport II
Problem Set 6
Due via Gradescope, 11:55 PM 3/2/23

1). In lecture we solved for the thermal entrance region Nusselt number for flow through a pipe where we had a constant heat flux at the wall. Using the same approach (e.g., self-similar profile for a thin boundary layer near the wall) solve for the local Nusselt number for the constant wall temperature case in the thermal boundary layer development region.

a. Using the "flat earth limit" case, derive the equations and boundary conditions governing the temperature profile.

b. Show that it admits a self-similar solution in this limit, and using this determine the local Nusselt number to within an unknown constant (e.g., the solution to the transformed self-similar DE).

c. Solve the DE using the shooting method (or do it analytically if you wish!), and compare your result graphically to the separation of variables solution provided in the problem of the day notes.

2). The local Nusselt number depends on both the boundary conditions and the geometry. For the case of constant wall heat flux, solve for the asymptotic Nu for laminar flow in a channel of width 2b (e.g., $u_x = 3/2\ U\ (1-y^2/b^2)$). Remember that this doesn't require a separation of variables solution to get an exact answer!

3). As always, there are strong analogies between unsteady conduction in solids and unidirectional convective heat transfer. This is particularly true if we have a flat velocity profile such as you would get if you had perfect wall slip (e.g., $u_z = U$ rather than our usual parabola). Using this analogy, determine the asymptotic Nu for a flat velocity profile for flow in a tube with constant wall heat flux. (Hint: once you've made the analogy it becomes pretty similar to a problem you've done before!)

4). Consider again flow through a channel of width 2b, this time with a flat velocity profile $u_x = U$ rather than the parabolic velocity distribution corresponding to Poiseuille flow. This makes it possible to solve the separation of variables problem analytically! For the case of a constant wall temperature (e.g., what was discussed in the problem of the day) determine the asymptotic value of Nu from the lead eigenfunction. Note that you don't even have to get the coefficient, as that should cancel out as we are limiting ourselves to a single eigenfunction.

Note: The results for all these problems are given in Table 14.2-1 of BS&L…