214

So far we've looked at conductive & convective transport - now we look at radiation

This is Energy transmission via electromagnetic radiation, propagating w/ velocity $c \equiv 3 \times 10^{10}$ cm/s across space, even in a vacuum!

Sometimes this is the largest source of energy transport!

What is electromagnetic radiation?

When a molecule is heated it moves into a higher energy excited state.

low temps, just rotation

higher temps, vibration

plasma temps = ionization

Transition is accomplished by thermal interaction or by absorption of radiation.

Transition to lower energy state is by thermal interaction or emission of radiation

Radiation is quantized: A transition of energy ΔE emits a photon of energy

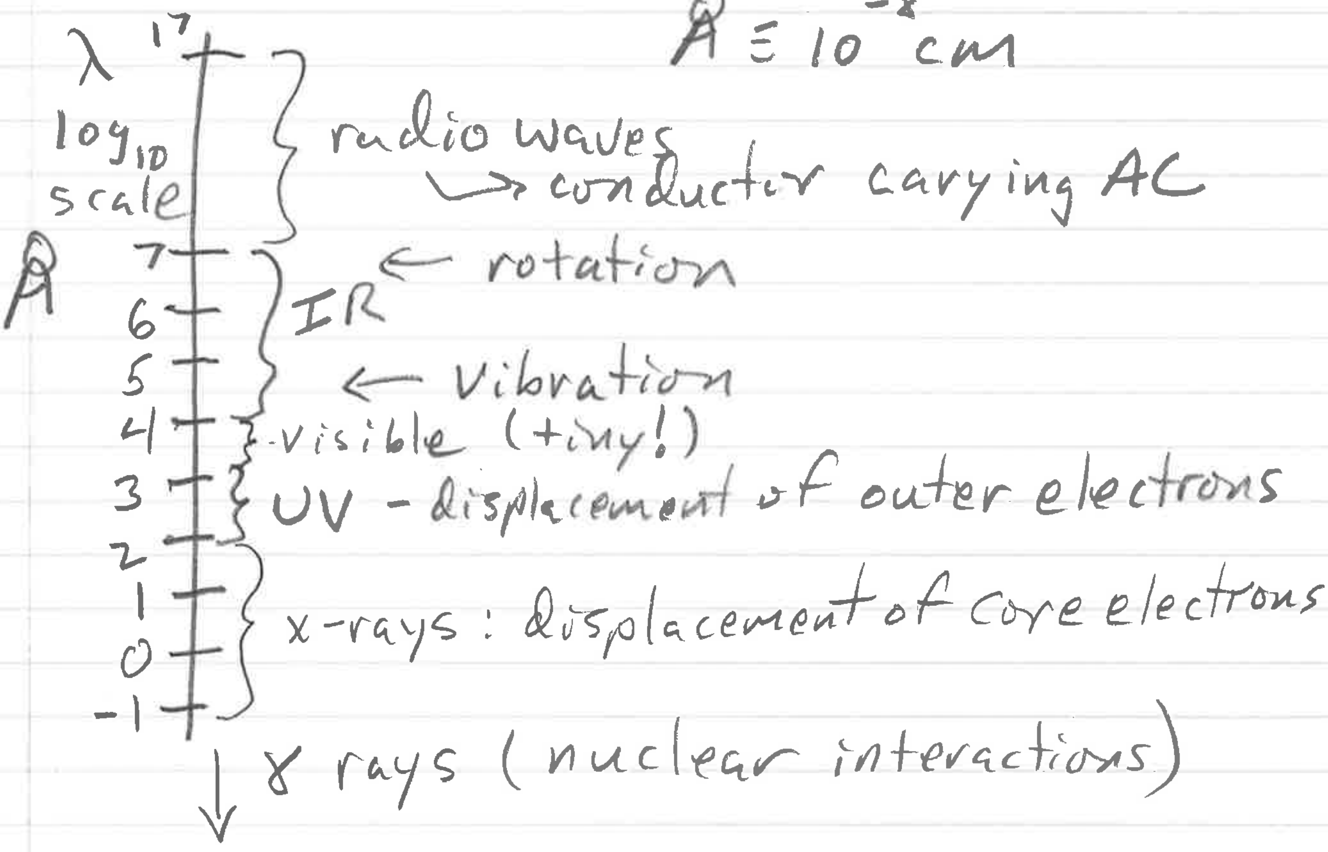
$\Delta E = h\nu$ ← frequency of radiation

↳ plank's constant

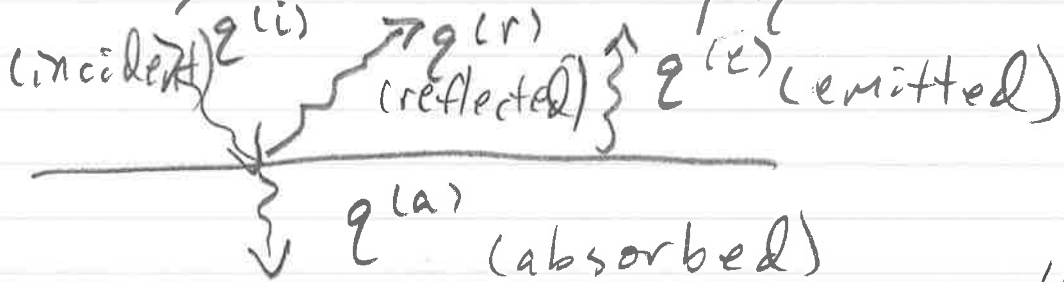
The wavelength of radiation is $\lambda = c/\nu$

There is a continuous spectrum of ΔE Energy levels!

$\text{Å} \equiv 10^{-8}$ cm



To examine the heat flux from radiation, consider an opaque solid:



we <u>define</u> absorptivity $a \equiv \dfrac{q^{(a)}}{q^{(i)}}$

the absorptivity <u>is</u> a $f^{\underline{n}}$(angle) <u>and</u> ν.

By definition $a \leq 1$ for all ν

we have an ideal gray body:

$a_\nu < 1$ <u>but</u> indep of ν

If $a_\nu = 1$ then it is a <u>black body</u> which absorbs <u>all</u> incident radiation.

A black body <u>emits</u> the largest flux of thermal radiation at all frequencies!

We can define an emissivity e:

$$e \equiv \frac{q^{(e)}}{q_b^{(e)}} \quad \& \quad e_\nu \equiv \frac{q_\nu^{(e)}\, d\nu}{q_{b\nu}^{(e)}\, d\nu}$$

blackbody

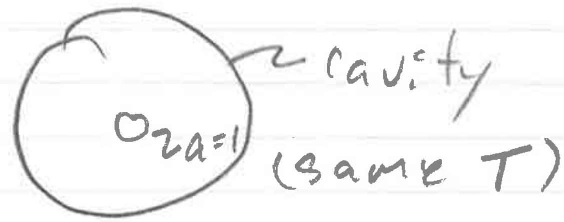
at frequency ν

Now we prove that $e \equiv a$!

Suppose we have an evacuated cavity w/ isothermal walls. At equilibrium there is no net exchange of energy between the cavity (filled w/ radiation) and the walls. Thus, the energy distribution of the cavity radiation is a function of T alone - wall composition doesn't matter!

It is also isotropic and unpolarized.

OK, now put a black body in the cavity at the same temp.



The black body <u>absorbs</u> the cavity radiation: $q_b^{(a)} = q^{(cav)} \equiv q_b^{(e)}$

and has to <u>emit</u> the same!

$\therefore$ Cavity radiation $\equiv$ black body radiation.

If you put a gray body in you get

$$q^{(a)} \equiv a\, q_b^{(e)} = e\, q_b^{(e)}$$

because no change in T!

$\therefore$ $a = e$

<u>and</u> at all ν, $a_\nu = e_\nu$

emissivity $\equiv$ absorptivity

This is known as <u>Kirchoff's</u> <u>Law</u>

Ok, how does $q_b^{(e)}$ depend on temperature?

Let's look at cavity radiation. Consider it as a gas made up of photons w/ energy $h\nu$ and momentum $h\nu/c$. Because it is isotropic the energy density is:

$$u^{(r)} = \frac{4}{c} \varrho_b^{(e)} \qquad \text{(energy/volume)}$$

The momentum of the photons exerts a pressure on the walls

$$p^{(r)} = \frac{1}{3} u^{(r)}$$

(This pressure is what drives light sails in space!)

The internal energy of our gas is

$$U = V u^{(r)}$$

From thermo:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Thus:

$$u^{(r)} = \frac{T}{3}\frac{\partial u^{(r)}}{\partial T} - \frac{u^{(r)}}{3}$$

or $\dfrac{\partial \ln u^{(r)}}{\partial \ln T} = 4$ so $u^{(r)} = bT^4$

$b \to$ some const.

Thus, $q_b^{(e)} = \sigma T^4$ (Stefan-Boltzmann Law)

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \, {}^\circ K^4}$$

You get the <u>same thing</u> from quantum theory! If photons obey Bose-Einstein statistics then you get Plank's Distribution Law:

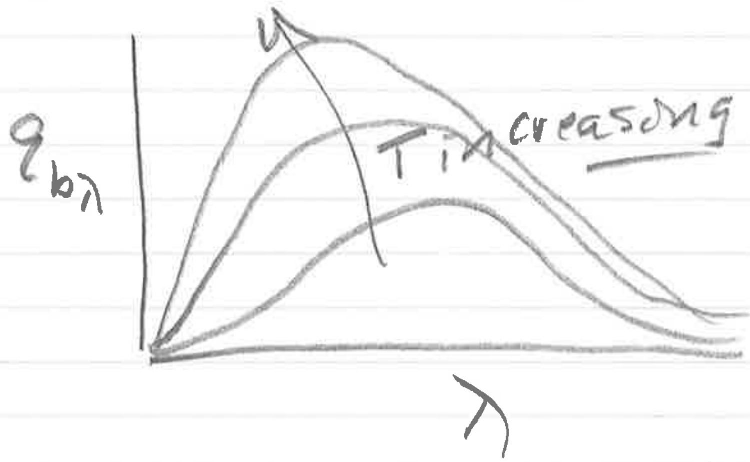
$$q_{b\lambda}^{(e)} = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1}$$

Which, integrated over λ yields:

$$q_b^{(e)} = \underbrace{\left(\frac{2}{15}\frac{\pi^5 k^4}{c^2 h^3}\right)}_{\sigma} T^4$$

221

What does this distribution look like?

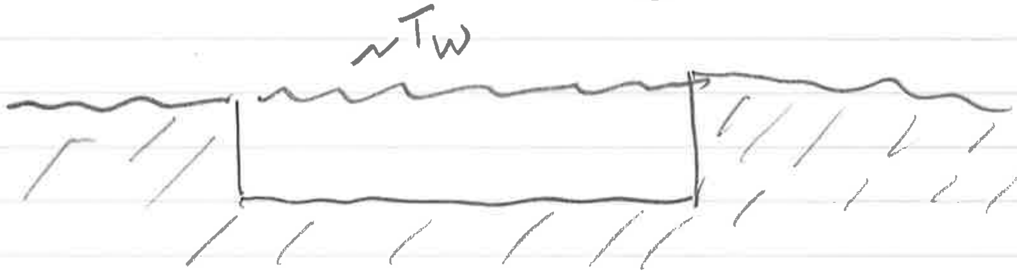


the max in $q_{b\lambda}$ shifts to shorter wavelengths (higher ν) w/ inc. T

For the sun the max is visible (green)

Ok, now let's solve a problem! (ex 16.5-3)

Suppose you put a pan of water out in the desert at night (insulating its bottom!). At what air temp. will it freeze? T_a



For dry still air you have a balance between natural convection to the surface & thermal radiation away!

$$\therefore \quad q = h(T_a - T_w) = e\sigma T_w^4 - e\sigma T^4$$

but $T \approx 0$

clear dry night is transparent (night sky!) to thermal radiation & space is cold

Ignore back radiation!

What's h? For a horizontal plane it's roughly $h = 1.3\,(T_a - T_w)^{1/4}$ where h is in $\frac{W}{m^2\,{}^\circ K}$ and T is ${}^\circ C$ (or ${}^\circ K$)

so: $\quad 1.3\,(T_a - T_w)^{5/4} = e\sigma T_w^4$

$$\therefore \quad T_a = T_w + \left(\frac{e\sigma T_w^4}{1.3}\right)^{4/5}$$

Now at freezing $T_w = 273\,{}^\circ K$

and $e \approx 0.95$ (black body at thermal λ)

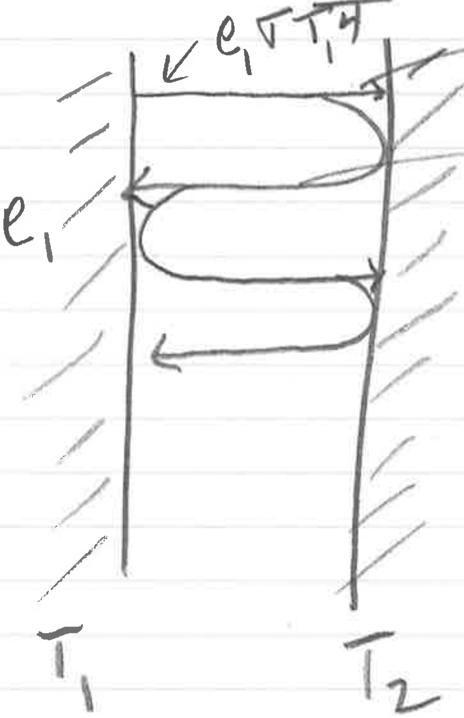
close, anyway!

$$\therefore T_a \cong T_w + \left(\frac{0.95\,(5.67\times 10^{-8})(273)^4}{1.3} \right)^{4/5} \quad 223$$

$$= T_w + 78^\circ K \equiv 78^\circ C!$$

This is unrealistic because you would get gain through your insulation, but it does show why the desert is cold at night (and why clouds keep it warmer!)

Ok, how about energy exchange between two gray planes?



absorbed $= e_1 e_2 \sigma T_1^4$

reflected back $= e_1 \sigma T_1^4 (1-e_2)$

So energy emitted by plane 1 gets both absorbed by (2) and reflected back to (1)

Thus, the energy transmitted from (1) to (2) is:

$$e_1 e_2 \sigma T_1^4 \sum_{i=0}^{\infty} (1-e_1)^i (1-e_2)^i$$

$$= \frac{e_1 e_2 \sigma T_1^4}{1 - (1-e_1)(1-e_2)} = \frac{\sigma T_1^4}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

↳ summing series!

The energy from (2) to (1) is the same!

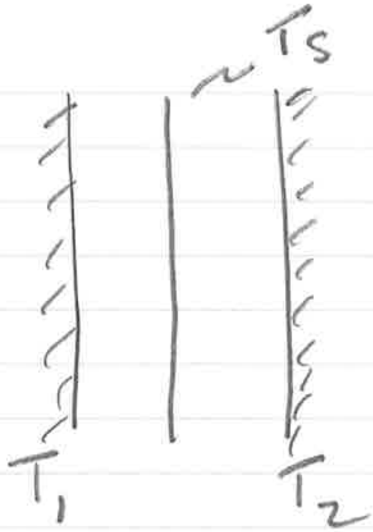
$$= \frac{\sigma T_2^4}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

∴ Net flux is $\mathcal{E}_{12} = \dfrac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1}{e_1} + \frac{1}{e_2} - 1\right)}$

We can use this to see the effect of a radiation shield.

Put a thin sheet w/ low e between two surfaces!

225



Now $q_{1s} = \dfrac{\sigma\left(T_1^4 - T_s^4\right)}{\frac{1}{e_1} + \frac{1}{e_s} - 1}$

and $q_{s2} \equiv \underbrace{q_{1s}}_{\text{at SS}} = \dfrac{\sigma\left(T_s^4 - T_2^4\right)}{\frac{1}{e_2} + \frac{1}{e_s} - 1}$

∴ solve for T_s and get:

$$q_{12}^{(w/s)} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{e_1} + \frac{1}{e_s} - 1\right) + \left(\frac{1}{e_2} + \frac{1}{e_s} - 1\right)}$$

If $e_1 = e_2 = e$

$$q_{12}^{(w/s)} = \frac{1}{2}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{e} + \frac{1}{e_s} - 1\right)}$$

so the ratio of w/s & without is:

$$\frac{\mathcal{E}_{12}^{(w/s)}}{\mathcal{E}_{12}} = \frac{1}{2}\frac{\left(\frac{2}{e}-1\right)}{\left(\frac{1}{e}+\frac{1}{e_s}-1\right)}$$

even if $e_s = e$, we have a factor of 2 improvement, and it is still reducing energy transp. if $e_s = 1$! (but a lot better if $e_s \ll 1$!)

You get a further reduction w/ multiple layers! In general, however, you would have conduction in the air between that limits the effect.

To finish off, let's look at spectral effects: what happens if two sources are at dif. T and e_ν is a function of ν?

This is exactly the greenhouse effect!

The sun is a black body at 5800°K and $q_\lambda\big|_{max}$ is $\lambda \sim 0.5\mu m$

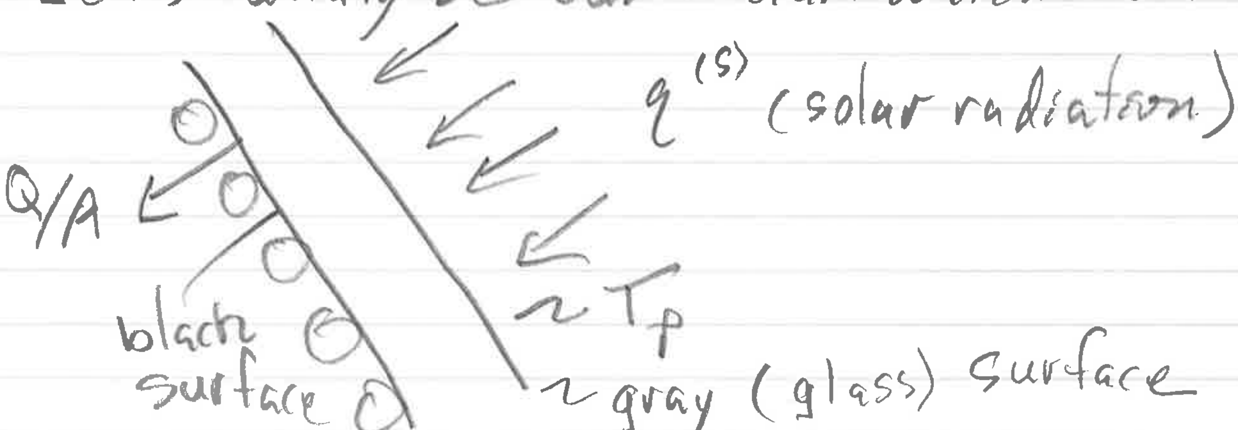
>99% of total E is for $\lambda < 4\mu m$

For a body at 300°K $\lambda_{max} \cong 10\mu m$ and >99% of total E is $\lambda > 4\mu m$!

We can maximize (thermal) solar gain if we cover our absorber w/ a sheet of glass!

glass is transparent to short wavelengths and a black body to long!

Let's analyze our solar water heater!



228

Ok, let's say we are boiling water!

$\therefore\ T_c = 373°K$

we have air at 300°K blowing over our glass plate w/ velocity of 10 mph, thus

$h_s \cong 20 \frac{W}{m^2\,°K}$ (approx external h)

we have natural convection between the collector & glass plate, take $h_i \approx 3 \frac{W}{m^2\,°K}$

we have an incident solar flux of

$q^{(s)} = 1350 \frac{W}{m^2}$

we can do an energy balance on the plate!

$$0 = \underbrace{e\sigma T_c^4}_{\text{abs. from collector}} - \underbrace{2e\sigma T_p^4}_{\text{emit radiation both sides}} + \underbrace{e\sigma T_a^4}_{\text{from surroundings}}$$

$$- \underbrace{h_s(T_p - T_a)}_{\text{losses to atmosphere}} + \underbrace{h_i(T_c - T_p)}_{\text{gain from collector}}$$

If we take $e=1$ then

$T_p = 323°K$

What is the energy gain of our collector?

$$\frac{\dot{Q}}{A} = q^{(s)} + e\sigma T_p^4 - (1-\rho)\sigma T_c^4 - h_i (T_c - T_p)$$

reflectivity of top plate (say $\rho = 1-e$)

So if $e = 1$ then $\frac{\dot{Q}}{A} = 770\ \text{W/m}^2$ (about half incident $q^{(s)}$)

We can do better w/ low emissivity coatings! A "low e" Fe_2O_3 glass is commercially available.

Ideally $t = 1$ for $0 < \lambda < 2.5\,\mu m$

and $e = a = 0.25$ for $\lambda > 2.5\,\mu m$
$(t = 0)$

Plugging this in yields $T_p = 312\,°K$ and $Q/A = 1090\ \text{W/m}^2$

If $e = 0$ (perfect) then $T_p = 307\,°K$

and $\dot{Q}/A = 1220\ \text{W/m}^2$!