CBE 31358 Junior Lab

Statistics Quiz # 3

Due in class on 3/25/11

Note: This is open book and open notes, however
don't discuss this quiz with your classmates!

Answer the following questions with a **MATLAB PROGRAM** . Turn in both your source code and your output.

Radiocarbon dating is a technique which is used to determine the age of archeologically interesting carbon-containing samples. It relies on the curious fact that (at least until the first atmospheric atom bomb tests) there is an equilibrium amount of radioactive C14 in the atmosphere, so that the radioactivity of carbon containing life forms was approximately uniform until exchange with the biosphere stopped (e.g., a plant, shell or animal died, or at least no longer grew). After that point, the radioactivity decayed away until it was no longer detectable according to the radioactive decay formula:

$$N = N_0 e^{-\lambda t}$$

where N is the current activity, N_0 is the original activity at the time of sequestration, λ is the decay constant of the material, and t is the age. For C14 the decay rate $\lambda = \ln(2)/5730$ in units of 1/yr. In the 1950's Willard Libby demonstrated that by measuring the radioactivity of an old sample, and by knowing both the initial activity (obtained from a reference standard) and the half-life, you could get the age of a plant, potsherd, bone, or shell. He was awarded the Nobel Prize in 1960 for this work. A nice discussion of the technique (and additional sources of error, which we won't worry about in this problem) is given online at:

http://en.wikipedia.org/wiki/Radiocarbon_dating

In my distant past (high school science project, actually) I cooked up a radiocarbon dating process which I could do in my basement (mostly). Basically, it involved using strong acids to liberate CO_2 from shells, cleaning up the gasses by absorbing them into a 10M KOH solution, and then reacidifying this solution and reabsorbing the evolved CO_2 into a liquid scintillation coctail. The resulting solution was placed into quartz vials and analyzed using borrowed time on some liquid scintillation spectrometers in the D.C. area. To calculate the age of a sample, you needed three measurements: The activity (decay rate, measured in cpm: counts per minute) of the sample S, the activity of a standard S_0, and the activity of a background sample B (made the same way as the sample and standard, but using marble (from which all C14 had decayed away) as the source of CO_2). This latter accounted for the not inconsiderable level of background radiation (the sun is a pretty fierce nuclear reactor - think about that next time you get a sun-tan). The ratio N/N_0 needed to calculate the age of the sample is just:

$$N/N_0 = (S - B) \;/\; (S_0 - B)$$

Of course, the measured values of S, S_0, and B have errors, which depend on the time the samples are left in the liquid scintillation spectrometer. For radioactive decay, the error in count rate is governed by Poisson Statistics, such that the standard deviation in the number of counts is equal to the square root of the number of counts. The number of counts is just the count rate multiplied by the observation time T. The variance in the count *rate* B, say, would thus be B/T.

Now for the problem:

For my experiments, typical values of activity were S_0 = 40cpm, S = 35cpm, and B = 32cpm.

1) Using these values and the formula given above, what is the apparent age of the sample?

2) If the observation time T for each of the vials was 400 minutes, estimate the standard deviation of the calculated age.

3) Note that, for large uncertainties, t is *not* a normally distributed variable! This is because of the non-linear dependence of t on S, S_0, and B. The ratio $(S-B)/(S_0-B)$ is much closer to being normally distributed. Using this, estimate the 95% confidence interval for the age of the sample.

4) Estimate the value of T necessary for this confidence interval to be less than 400 years. (Hint: how does the standard deviation of the age decrease with time?)