CBE 31358 Junior Lab

Statistics Quiz # 4

Due in class on 4/8/11

Note: This is open book and open notes, however
don't discuss this quiz with your classmates!

Answer the following question with a **MATLAB PROGRAM** . Turn in both your source code and your output.

In a classic paper (cited 182 times at last count) Karnis, Goldsmith, and Mason (1966) observed that the velocity profile of a suspension of particles flowing through a tube was blunted from the parabola expected (and observed) for a Newtonian fluid. While they recognized that the obvious explanation for the blunting was an inward migration of particles yielding a higher concentration (and thus viscosity) near the tube axis, they asserted that no such concentration variation occurred. Their data is given below (it's the 33% volume fraction experiment for which blunting was observed). Their measurement procedure was via a tracer sphere technique. A small fraction of the particles were marked, and the number of marked particles sharply in the focus of the microscope they used for visualization were counted in each of four radial bins. Because the fraction of tracers was very small (the rest of the spheres in the suspension were transparent so they could see inside), the number of tracers observed was subject to Poisson statistics, just like radioactive decay. We shall take the data in the last column to test the assertion that the concentration profile was really uniform.

Using dimensional analysis arguments, current models suggest that the dimensionless concentration gradient r dc/dr should be a function of concentration alone, at least for small particles in fully developed flows. To analyze the above data, take the concentration at the center of each bin to be the average concentration of each bin (e.g., $4*N/N_t*0.33$). Then take the concentration difference between neighboring bins and divide by the distance between the bin centers to get the gradient dc/dr. Finally, multiply by the midpoint between the pairs of bins to get r dc/dr. In this way you will get three measures of this ratio. Calculate the average of these to get an experimental estimate of r dc/dr at 33%.

OK, now for the questions:

1) What is the magnitude of r dc/dr (together with the 95% confidence interval) estimated from the last column of Table IV below? Is the deviation from the expected value of -0.135 (e.g., Ramachandran and Leighton, 2008) statistically significant at the 95% confidence level?

2) Measurements by a number of investigators using MRI techniques have subsequently shown that the blunting really was caused by concentration variations across a tube (e.g., Hampton, R. E., Mammoli, A. A., Graham, A. L. & Tetlow, N. 1997; Altobelli, S. A., Givler, R. C. & Fukushima, E. 1991 among many others). Briefly comment on this relative to your answer to question 1.

TABLE IV

CONCENTRATION PROFILES OF TRACER SPHERES IN TUBE FLOW

$b/R_0 = 0.039$; $R_0 = 0.4$ cm.;
$Q = 3.56 \times 10^{-2}$ cm.^{3} sec.$^{-1}$.

$c = 0.17$			$c = 0.33$		
Range of r (*cm.*)	No. of tracer particles N		Range of r (*cm.*)	No. of tracer particles N	
	$t = 0$	$t = 4$ hr.		$t = 0$	$t = 4$ hr.
0.334–0.400	62	57	0.300–0.400	71	71
0.267–0.333	56	58	0.200–0.299	72	69
0.200–0.266	56	51	0.100–0.199	76	76
0.134–0.199	55	65	0.00 –0.099	80	84
0.067–0.133	63	64			
0.000–0.066	61	66			
Total	353	361		299	298

Hint: Don't forget that N_t is also determined from adding the individual N's together, and thus isn't an independent variable. This problem is quite easy if you write a function that takes in the "N" values as an array (be sure to put them in the right order!), and spits out the calculated average r dc/dr using the algorithm above. Since you know the matrix of covariance of the "N" values (diagonal, assuming Poisson statistics), you can calculate the dependence of the function on each N and propagate the error forward using the standard formula!

Aside: It was largely because of this paper that for about 15 years investigators believed that the concentration profile of a suspension in a tube was uniform. This was fortunate for me, because that's why the problem was still around for me to solve in my PhD thesis in the early '80s (e.g., Leighton and Acrivos, 1987). You can get at all of these papers using Google Scholar from any computer on the ND network.