CBE 31358 Junior Lab

Statistics Quiz # 5

Due in class on 4/15/11

Note: This is open book and open notes, however
don't discuss this quiz with your classmates!

Answer the following question with a **MATLAB PROGRAM** . Turn in both your source code and your output.

In our laboratory we are currently trying to measure the dynamics of a dilute suspension of particles undergoing shear in a parallel-plate device by observing the motion of tracer particles. Experimentally, we look down through the suspension and observe the r,θ position of particles which are in an observation window $-\Delta\theta < \theta < \Delta\theta$ and $R_1 < R < R_2$. The problem is figuring out which particle in one image corresponds to a particle in subsequent images. The images are all taken 1 second apart, sufficiently close that they move in the θ direction only a small distance, and very little distance in the radial direction.

The velocity profile for parallel-plate flow is $u_\theta = \Omega\, r\, z/h$ where Ω is the angular velocity of the upper plate, r is the radial position, z is the vertical coordinate, and h is the gap width. Thus, by measuring the displacement in the θ direction in successive frames (or in a sequence of frames) we can get an estimate of the vertical position.

It is fairly straightforward to track particles while they are in the field of view - you just look for particles in successive images that are at the same r, θ position (to within some trust interval). The problem is linking up the particle with previous observations after it leaves the window and it comes around again. This is because there is uncertainty in the vertical position (and hence its circuit time).

OK, now for the questions:

1) From the time series of position identifications below, calculate the dimensionless time Ωt_0 at which the particle is at $\theta = 0$, as well as its radial position and dimensionless vertical position z/h.

2) Calculate the random uncertainty (standard deviation) in these values.

3) Extrapolating forward, calculate the expected dimensionless time Ωt_1 at which θ should reach 2π.

4) Calculate the uncertainty in this time due to parameter estimation in (1). Note that you also have to include the uncertainty in Ωt_0! This, combined with the uncertainty in r and z/h obtained in (2), tells you the region over which you should look for the particle in tracks calculated from later images.

The data is as follows:

$\Omega = 0.05$ rad/s
$h = 1$ cm

and the observations:

time	r	θ
0	7.2950	-0.0329
1.0000	7.2231	-0.0130
2.0000	7.2607	0.0056
3.0000	7.2486	0.0155
4.0000	7.2891	0.0149
5.0000	7.2762	0.0363
6.0000	7.2456	0.0660
7.0000	7.2019	0.0805
8.0000	7.2821	0.0814
9.0000	7.2445	0.1098

Note: The "real" problem is actually quite a bit more complex than the version given above, since in addition to the deterministic motion you are worrying about, the particle also diffuses in the z direction, leading to additional dispersion in the θ direction (a fairly large term). This broadens the window over which the particle should appear in later images quite substantially - but it is also much trickier to calculate...