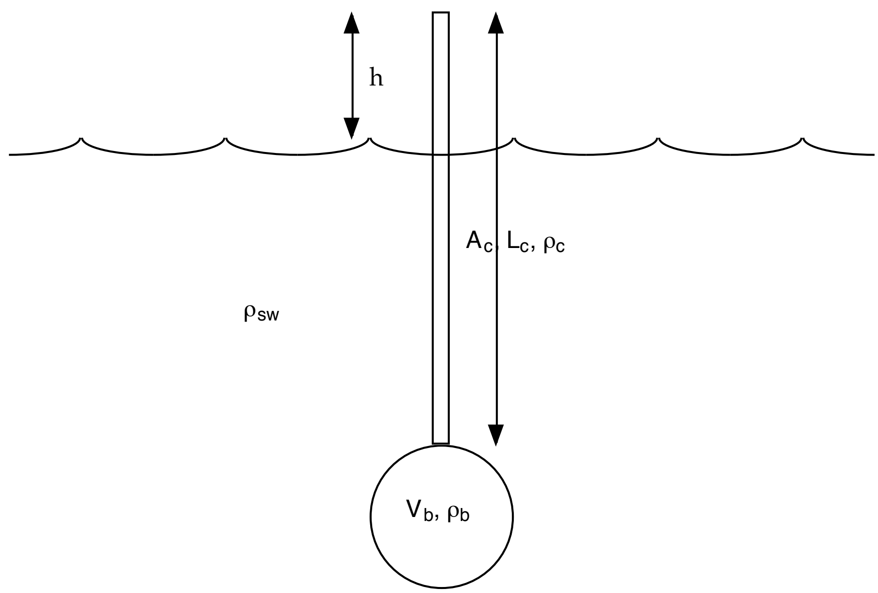
**CBE 30355 TRANSPORT PHENOMENA I**

First Hour Exam
10/2/12

**This test is closed books and closed notes**

**Problem 1.** (10 pts) In this problem we design a *hydrometer*, a simple device for measuring the density of a liquid, usually used in determining the salt concentration. A simplified version is depicted below, and consists of a bulb of volume $V_b$ and density $\rho_b$, and a column of length $L_c$, cross sectional area $A_c$, and density $\rho_c$. Given this, determine the relation between the height h the column sticks up out of the water as a function of $\rho_{sw}$ (the density of the salt water solution we are interested in) and the other parameters in the problem.




**Problem 2.** (15 pts) Unidirectional Flows: Consider the system depicted below. A closed end cylinder of radius R is filled with a fluid of density $\rho$ and viscosity $\mu$. A thick rod of radius a, length L and density $\rho_s>\rho$ settles downwards under the action of gravity with velocity U, displacing fluid. If the length L>>a, the flow in the gap may be regarded as unidirectional in the z-direction.

a. If the rod velocity is U, develop an integral expression for the fluid flow rate in the gap between the rod and the cylinder.

b. Write down the differential equation and boundary conditions (integral and otherwise) governing the velocity distribution in the gap.

c. Show how you can calculate the settling velocity U in terms of the density difference $\rho_s$-$\rho$. You do not need to evaluate the constants of integration (they are a bit messy), but make it sufficiently detailed that it is clear that you know how to get the final result.

The equations of motion in cylindrical coordinates are given by:

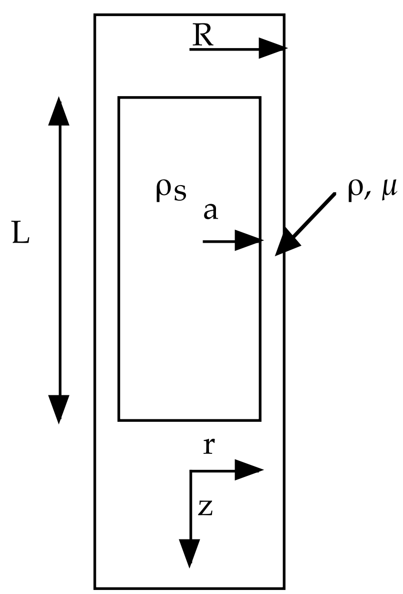
$$r: \quad \rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\phi^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \phi^2}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\phi}{\partial \phi}\right]+\rho g_r$$

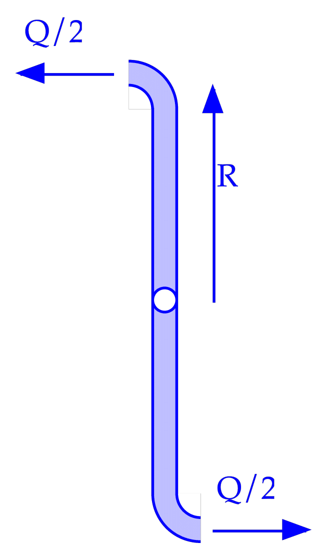
$$\phi: \quad \rho\left(\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_\phi}{\partial \phi}+u_z\frac{\partial u_\phi}{\partial z}+\frac{u_r u_\phi}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \phi}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\phi}{\partial \phi^2}+\frac{\partial^2 u_\phi}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \phi}-\frac{u_\phi}{r^2}\right]+\rho g_\phi$$

$$z: \quad \rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_z}{\partial \phi}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \phi^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z.$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial(\rho u_\phi)}{\partial \phi}+\frac{\partial(\rho u_z)}{\partial z}=0.$$

$$\tau_{rz}=\tau_{zr}=\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)$$




**Problem 3.** (10 pts) Consider the sprinkler depicted above. The radius of the arms is R, the area of the jet outlet is A (for each arm), and the total flow rate of water is Q (e.g., Q/2 for each arm). Using an integral momentum balance, determine the torque exerted on the center of the sprinkler if the arms are stationary.

**Problem 4.** (15 pts) Short Answer / Index notation / Additional Readings

1. Order according to their date of death (oldest -> most recent):
   A. James Clerk Maxwell
   B. Hero of Alexandria
   C. Osbourne Reynolds
   D. G. I. Taylor

2. Crooke's Radiometer works because of:
   A. The momentum of light
   B. Thermal transpiration
   C. Maxwell said it should
   D. Hot gas on the black face of the vane

3. The most deadly aspect of volcanism is:
   A. Sediment flows
   B. Lava flows
   C. Pyroclastic flows
   D. Mudslides

4. The vorticity is defined as the curl of the velocity. Write down this defining relation using index notation.

5. What is the most general isotropic second order tensor?

6. (2 pts) Write down the continuity equation for a **compressible** fluid using index notation.

7. (2 pts) The symmetric part of the rate of strain tensor is usually given the symbol $e_{ij}$. What is its representation in terms of $u_i$? (Index notation, please!)

(2 pts each) Identify the physical mechanism behind the following terms, **and provide an example of where they would play a role**:

8. $\rho \underset{\sim}{u}$ (two interpretations!)

9. $\mu \dfrac{\partial^2 u_x}{\partial y^2}$

10. $\rho \dfrac{v_r v_\theta}{r}$