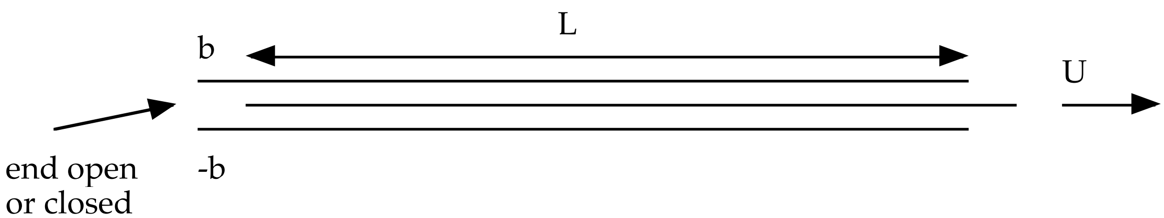
**CBE 30355** **TRANSPORT PHENOMENA I** **TTh 11:00**

**Second Hour Exam**

**Closed Books and Notes** **November 12, 2019**

**Note: Problems are not equally weighted!**

Problem 1). (20 points) Unidirectional Flows: A plate occupies the center of a slot of width 2b and length L (and extension into the paper W) as depicted below. Each side is independent, so you can calculate things just looking one side of the plate. The plate is pulled out with velocity U. The ratio b/L <<1 so you can assume unidirectional flow.




a. If the slot is open on both ends, no pressure gradient can be produced. For this case, calculate the velocity profile in the slot and tangential force on the plate.

b. If the slot is closed at one end the net flow goes to zero and a pressure gradient is produced, significantly increasing the force on the plate (a much more efficient damper!). Calculate the velocity distribution and force on the plate for this case.

c. For the closed end case there is also a normal force on the walls of the slot (it tends to collapse towards the plate if not supported). Calculate this force. Just use the top wall of the slot, the force on the bottom would be the negative of this.

Problem 2). (20 pts) Scaling/Unidirectional flows: Consider a vertical straw of length L and radius a containing a liquid with viscosity $\mu$ and density $\rho$. The liquid is initially at rest (e.g., you have put your finger over the end so that no flow occurs). At time t=0 you take your finger off the end and gravity causes the fluid to flow. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time $T_d$ required to empty the straw in both inertia and viscous dominated conditions - e.g., how long does it take for h to reach zero.

a. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: **unsteady**, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Render the equations and boundary conditions dimensionless, scaling the one term which you -know- has to be there to be O(1). Don't forget the equation for h!

c. Scale the equations and boundary conditions for the case where inertia is the dominant physical mechanism, and determine the (unknown) characteristic drainage time $t_c$ in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

d. Scale the equations for when viscous effects are dominant, and determine the new (unknown) characteristic drainage time $t_c$ in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

e. If the straw is 20cm long and the radius is 2mm, which limit is more appropriate for water? Show your work!

The equations of motion in cylindrical coordinates are given below (you don't need all of these for this problem!):

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

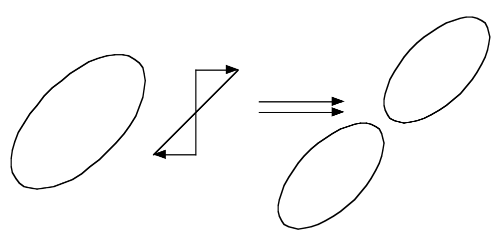
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\,\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 3). (10 points) Dimensional Analysis: A popular way to make water-in-oil emulsions (basically suspensions of really tiny drops) is by subjecting a mixture of oil and water to high shear rates. The shear stresses in the oil tear apart the drops of water until they reach the desired size. It is observed that the final drop radius a is a function of the surface tension $\Gamma$, the shear rate $\dot{\gamma}$, the oil viscosity $\mu_f$ and the drop viscosity $\mu_d$.



a. Form the dimensional matrix of the parameters governing this problem, and determine the number of dimensionless groups involved.

b. If $\Gamma = 7$ dyne/cm, $\mu_f = 70$cp, and the drop phase is water ***make an estimate*** of the oil phase shear rate required to achieve a drop radius of 50$\mu$m. Hint: it is the viscosity of the phase outside the drop which primarily controls the process!

Problem 4). (10 points) Dimensional Analysis in Ship Design: A 1:64 scale model of a ship is constructed for study of hull design in a towing tank.

a. If the velocity of the model is $U_m$, what would be the corresponding velocity of the full scale ship?

b. If the force required to move the model at this velocity is measured as $F_m$, what would be the corresponding required power output of the engines of the full scale ship, assuming 75% efficiency of the propellers (a reasonable value)?

c. What is the ratio of the Reynolds number of the model to that of the full scale ship? Why would this matter?

Problem 5). (20 points) Short Answer: *Very* briefly answer the following:

1) What does Stokes' Law describe, and where is it valid?

2) What is its mathematical expression in terms of parameters (with the constant too!).

3) What does the Froude number represent?

4) Estimate the Reynolds number of a 0.5cm diameter minnow swimming at 10cm/s.

5) What equation does a harmonic function satisfy (name or mathematical expression)?

6) What are the key assumptions which lead to Newton’s Law (of viscosity)?

7) What is the difference between strict and approximate dynamic similarity, giving one example?

8) What equation governs the streamfunction for 2-D flow at zero Re?

9) What is a Stokeslet?

10) What is the difference between quasi-parallel and unidirectional flow? Give an example of each.