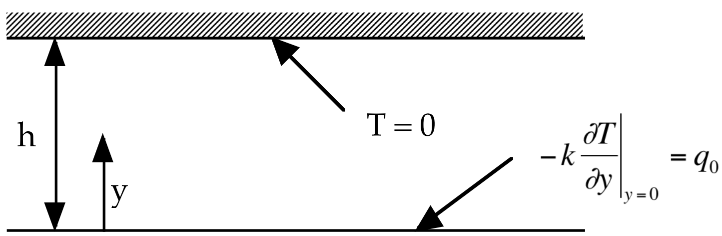
**CBE 30355 Transport Phenomena I**
**Final Exam**

**December 13, 2018**

**Closed Books and Notes**

Problem 1. (20 points) Unsteady Heat Transfer in Solids. A material with thermal conductivity k (and thermal diffusivity $\alpha = k/\rho C_p$, the heat transfer analog to the momentum diffusivity $\nu$) is sandwiched between two planes separated by a distance h. Initially the temperature of the entire system is at a reference value of zero, and the surface at y = h is maintained at this temperature for all time. For t > 0 there is a constant heat flux $q_0$ at the lower wall (y = 0). Here we examine the resulting temperature distribution.




a). The starting point. Write down the governing differential equation and boundary conditions valid for all times. Remember that heat transfer in solids *is mathematically identical* to unidirectional flow problems!

b). Render the problem dimensionless in the long time limit and determine the asymptotic temperature distribution at long times.

c). For very short times the length scale isn't the gap width, but instead should be some boundary layer thickness δ. By scaling the equations in this limit, how do δ and the temperature scale with $t_c$?

d). Using either the results of part c, or via simple affine stretching, show that the short time boundary layer problem admits a similarity solution. Give the similarity rule and the similarity variable in canonical form, and determine the time dependent temperature of the lower wall to within some unknown multiplicative constant (e.g., the solution of the transformed differential equation that you don't have time to get).

e). The solution in part d will break down after some time t. When will this occur and why?

Problem 2. (20 points) Unidirectional Flows: In the annual University sponsored health screenings for faculty and staff, technicians run a blood test to measure cholesterol and glucose levels. What they do is poke a hole in your finger to get a drop of blood and then draw the blood into a capillary which is then inserted into the analysis machine. The blood is drawn into the capillary by surface tension as depicted below. If the surface tension of blood is $\Gamma = 56$ dynes/cm, the viscosity of blood is 4cp, the radius of the capillary is $a = 50\mu$m, and the length $L = 5$cm, how long does it take for the tube to fill? Recall that the capillary pressure for a fully wetted *circular* tube of radius a is $\Delta p_{cap} = 2\Gamma/a$.

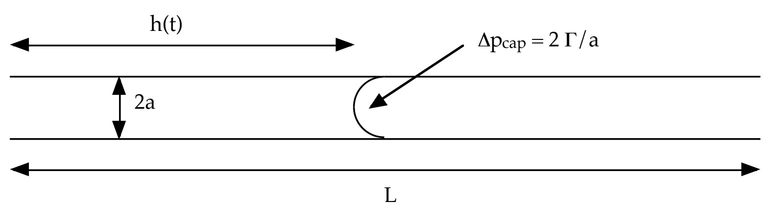
(Hint: Inertial effects are negligible for this problem, and the pressure differential is applied only over the filled length h of the capillary, which changes in time at a rate equal to the average axial velocity!)

a. Write down the governing equations and boundary conditions, as well as the equation governing the time dependent filled length h.

b. Render the equations dimensionless to determine how the filling time depends on the parameters of the problem.

c. Solve the problem to determine when the capillary is completely filled in terms of the parameters of the problem.

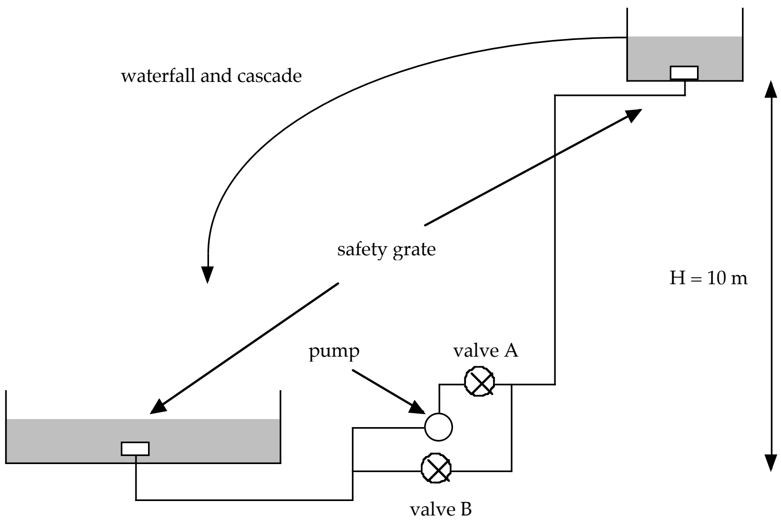
d. Plug in the numbers to get the final numerical value.




$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

Problem 3. (20 points) High Re Flow/Pump Curves: A very attractive part of many public gardens is the water feature (it's my favorite part, anyway). Because most of these gardens aren't blessed with a naturally occurring source of flowing water, the features are usually driven by a pump. Here we size such a system.

Consider the garden feature depicted below (the picture is from the Royal Botanical Garden near Toronto). Water is pumped from a lower reservoir into an upper reservoir which then cascades in a waterfall and a creek back into the lower reservoir. The piping network consists of 50m of 3" ID pipe, and begins and ends with a couple of safety grates (K factor of 1.5 each). The valves are gate valves which can be set either for pump operation (valve A open, valve B closed), or for draining the upper reservoir (valve A closed, valve B open). Note that a centrifugal pump usually redirects the flow, so you don't have an extra elbow for it in the diagram below.




a. It is desired to run the waterfall at 15 liters/s. What is the energy requirement for the pump CP-80i to provide this flow? Operating costs are mostly energy, which costs \$0.15/kwhr. What is the daily cost of operating the waterfall?

b. How far above the lower pond can we site the pump before "bad things happen"? Assume you've got about 10m of pipe leading to the pump.

c. It is proposed to reduce operating costs by using 4" ID pipe rather than 3" ID pipe. The larger pipe currently costs \$2/ft more. Remember how velocity and losses scale with pipe diameter! Using this, *estimate* (e.g., use the same friction factor, not a terrible approximation) how long reduced operating costs would take to pay for the larger diameter pipe.

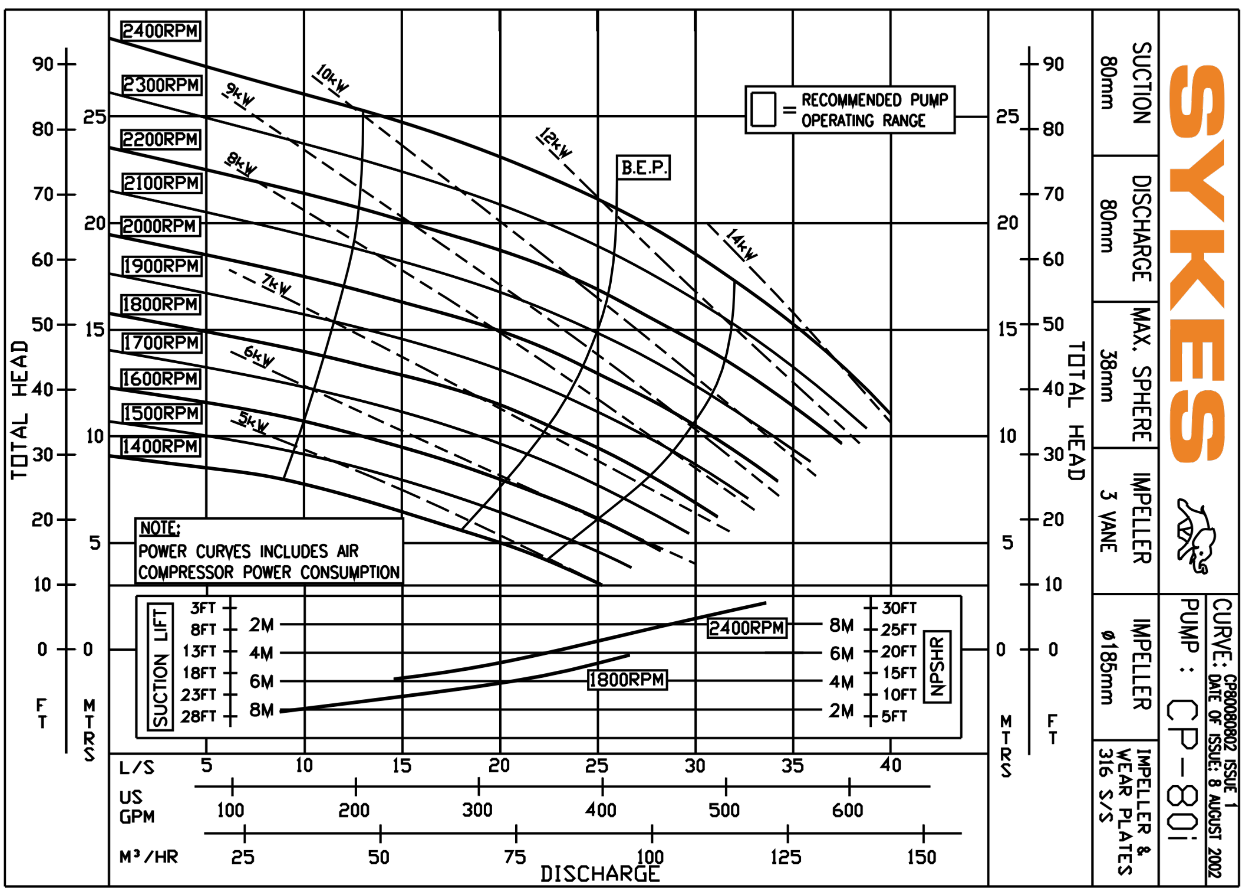
d. The upper reservoir has a volume of 5,000 liters. If we close valve A and open valve B, *estimate* the length of time for the reservoir to drain (with the 3" pipe).

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \mathrm{Re} > 3000$$

| Fitting | K value |
|---|---|
| safety grate | 1.5 |
| gate valve (open) | 0.15 |
| 90° elbow | 0.70 |
| T (straight through) | 0.4 |
| T (through side) | 1.5 |



Problem 4). (20 points) Short Answer:

1. What is the Coanda Effect and where does it arise?

2. What is the Magnus Effect and where does it arise?

3. Which (if any) of the following ***must*** be continuous at a fluid-fluid interface?

A. Shear rate
B. Heat flux
C. Mass flux
D. Velocity

4. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

5. For a shear stress of 25 dynes/cm$^2$ in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

Briefly identify the following equations **and a problem where it would play a role**:

6. $D_0 = \frac{kT}{6\pi\mu a}$

7. $U_s = \frac{2}{9}\frac{\Delta\rho g a^2}{\mu}$

8. $\nabla^4 \psi = 0$

9. $Q = \frac{-\pi}{8}\frac{\Delta P}{L}\frac{R^4}{\mu}$

10. $p + \frac{1}{2}\rho u^2 + \rho g h = Cst$