CBE 30355 Transport Phenomena I
Final Exam

December 13, 2021

Closed Books and Notes

Problem 1. (20 points) Thermal boundary layers. Consider the system depicted below. The fluid velocity is just **unidirectional simple shear flow in the x-direction**. The fluid enters with a temperature of zero (e.g., we've already subtracted off some reference temperature), and gets heated by a wall heat flux that is **linearly increasing** as we travel down the plate, e.g., $\mathbf{q_w} = \boldsymbol{\lambda}\, \mathbf{x}$ where λ is some constant. The thermal energy equation is given by:

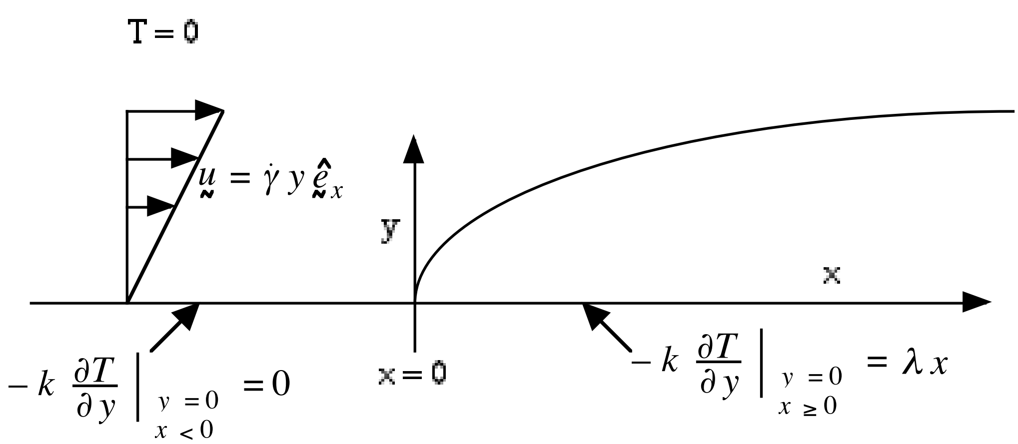
$$\frac{\partial T}{\partial t} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla} T = \alpha \nabla^2 T$$

with boundary conditions:

$$-k \frac{\partial T}{\partial y}\bigg|_{\substack{y=0\\x\geq 0}} = \lambda x \quad , \quad -k \frac{\partial T}{\partial y}\bigg|_{\substack{y=0\\x\leq 0}} = 0 \quad , \quad T\big|_{y\to\infty} = 0$$

a). Render the governing equation and boundary conditions dimensionless using a length scale L in the x-direction, and determine the conditions under which we can expect a thin boundary layer in the y-direction (e.g., how big does L have to be?). You may assume steady-state, with no variation in the z-direction.

b). Show that the thermal boundary layer equations yield a self-similar solution, obtaining the similarity rule and similarity variable in canonical form. Using this, determine the temperature at the plate as a function of x to within some undetermined multiplicative constant. Note that you don't have to get the transformed ODE or solve it to do this!



Problem 2. (10 pts) Your classmates demonstrated film vapor condensation, fluid inertia, and the rather large pressure of the atmosphere by looking at how you can crush a soda can. Here we will try to do an estimate of just how fast water vapor can condense in a can. A soda can (volume 360cm^3, surface area of 226cm^2) will crush if the force on the sides exceeds 50 N. In the experiment, the can was filled with water vapor (a bit of liquid on the bottom was boiling), displacing all the air. The can was quickly turned over and immersed in ice water, and then interesting things happen. The following observations are made:

1) If the can is dipped in the ice water so that the opening is up (exposed to air) nothing happens.

2) If the hole on the top has a radius of 1 cm and the can is inverted, it implodes.

3) If the hole on the top is enlarged to a radius of 2 cm and the can is inverted in the ice water, nothing happens.

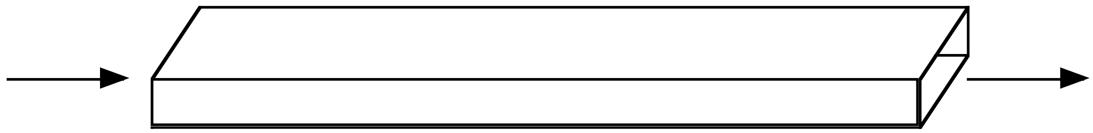
Using these observations and your knowledge of fluid mechanics, briefly explain what is going on in the three cases and estimate the time necessary for the water vapor to condense on the surface of the can. You could actually use this to get a film heat transfer coefficient, but I think we'll leave that for next year…

Problem 3. (10 pts) Food rheology is very important for all sorts of reasons. One desirable trait (for certain products) is a yield stress, the minimum stress at which a fluid will flow (zero for Newtonian fluids such as air, water, and honey). In class it was demonstrated how a tube, a pressure source, and a water manometer could be used to determine the yield stress of ranch dressing.

a. If the tube filled with the dressing is of radius a and length L, and the dressing first starts to flow when the height difference of the water in the manometer monitoring the pressure reaches a value h, develop a formula for the yield stress in terms of these parameters.

b. If the radius is 1mm, the length is 10cm and the height difference is 5cm, what is the quantitative value of this yield stress?

Problem 4. (10 points) Plane Poiseuille Flow: An important problem in bioengineering is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. To study this, a researcher has built a rectangular flow cell which is 100μm deep, 1mm wide, and 2cm long. In the test, cells adhere to the lower wall (the 1mm by 2cm surface) and their motion is observed. Initially they are stuck, but at a flow rate of 30 μl/min half the cells are torn away. Based on this, what is the critical shear stress associated with cell adhesion?



Problem 5. (20 points) You are tasked with using the pump CP-80i (pump curve attached) to pump water 10m uphill to fill a reservoir for irrigation purposes. The problem is that you only have a 10kW power supply for the pump and need to determine what flow rate you are going to get for the system. So:

a. If you have a really large diameter pipe for your system pipe losses will be negligible. In this case, what flow rate can you expect (really the upper limit) and what would the pump efficiency be?

b. The proposed piping network consists of 40m of 3″ schedule 40 PVC tubing (ID = 7.72cm), together with 6 90 degree elbows and a safety grate at the inlet with K value of 1.5. Given this, get a better estimate of the flow rate you can expect from the pump (still running at 10kW). Note that this would require a few iterations to get the exact value, so just show your work and try to get close in a short amount of time. Kick the iteration off by guessing that losses are about equal to the change in height (not a bad guess, actually) and using the appropriate flow rate from the pump curve. I'm looking for method here!

c. The overall efficiency of the system would be the useful work out (e.g., the potential energy gained by the water by pumping up hill) divided by the energy into the pump. Note that this is intrinsically lower than the pump efficiency. What is this overall efficiency based on the flow rate you calculate in b?

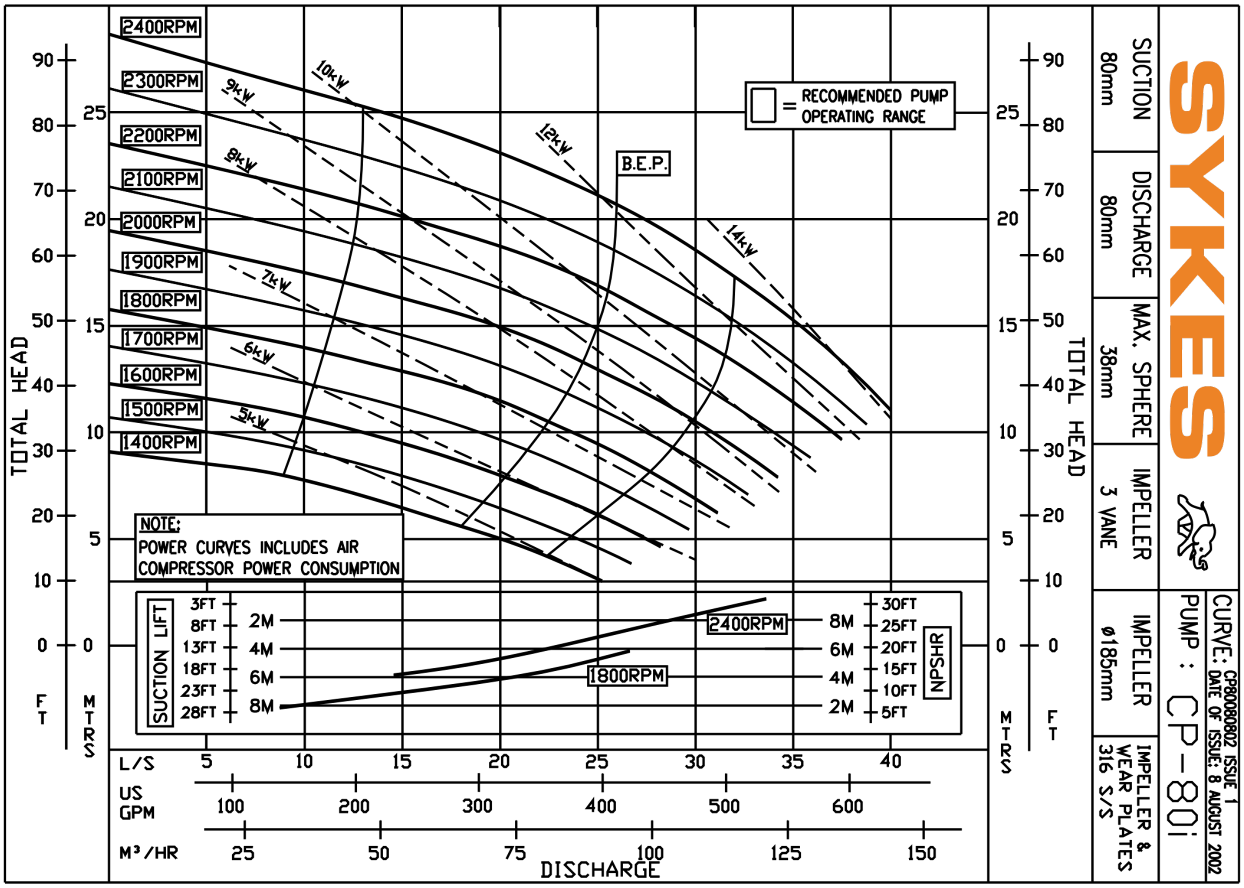
d. The piping leading to the pump from the source consists of two elbows, the safety grate, and 4 meters of pipe. Based on this, what recommendations can you make about the placement of the pump relative to the water source?

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \, f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \mathrm{Re} > 3000$$

Fitting	K value
sudden expansion	1.0
90° elbow	0.90
safety grate	1.5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7kW
6kW
5kW
12kW
14kW
B.E.P.
= RECOMMENDED PUMP OPERATING RANGE
NOTE:
POWER CURVES INCLUDES AIR COMPRESSOR POWER CONSUMPTION
TOTAL HEAD
FT
MTRS
SUCTION LIFT
3FT 8FT 13FT 18FT 23FT 28FT
2M 4M 6M 8M
NPSHR
30FT 25FT 20FT 15FT 10FT 5FT
8M 6M 4M 2M
2400RPM
1800RPM
L/S
US GPM
M³/HR
DISCHARGE
SYKES
SUCTION 80mm
DISCHARGE 80mm
MAX. SPHERE 38mm
IMPELLER 3 VANE
IMPELLER ø185mm
IMPELLER & WEAR PLATES 316 S/S
PUMP : CP-80i
CURVE: CP80080802 ISSUE 1
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