PROBLEM SET 1 CHEG 355 DUE 9/3/02

These first few problems should serve as a review for the vector calculus material you learned last semester and will use extensively this term.

1). Calculate the angle between the following pairs of vectors ($\mathbb{e}_x$, $\mathbb{e}_y$, $\mathbb{e}_z$):

a. (1,0,1), (1,0,0)

b. (1,0,1), (0,1,0)

c. (1,2,3), (3,2,1)

2). Calculate the following quantities (Note: X denotes the cross-product and · the dot product):

a. (1,0,1)·(1,0,0)

b. (1,0,1)·(0,1,0)

c. (1,2,3)X(3,2,1)

3). For the scalar potential function $\phi = (x^2+y^2+z^2)^2$ and the velocity vector field $\mathbb{u} = (y^2, z, x^2)$ calculate the following vector quantities:

a. $\mathbb{\nabla}\phi$; $\mathbb{\nabla}\cdot\mathbb{u}$

b. $\nabla^2\phi = (\mathbb{\nabla}\cdot\mathbb{\nabla})\phi$; $\nabla^2\mathbb{u}$

c. $\mathbb{\nabla}X\mathbb{u}$

where the boldface operator $\mathbb{\nabla} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

4. Prove that for an arbitrary scalar function ϕ:

$\mathbb{\nabla}X(\mathbb{\nabla}\phi) = 0$

5. This is <u>completely</u> optional (and not for credit - solve only if you like puzzles): Prove the following vector identity for the arbitrary vector $\mathbb{u}$:

$$\mathbb{\nabla}X(\mathbb{\nabla}X\mathbb{u}) = \mathbb{\nabla}(\mathbb{\nabla}\cdot\mathbb{u}) - \nabla^2\,\mathbb{u}$$

Hint: I usually solve this problem using *index notation* which is very useful for describing advanced transport problems. Detailed notes on index notation are available through the class website. We'll go over index notation in the first review session.

In this problem set all vectors are in outlined boldface type while scalars are in regular type.