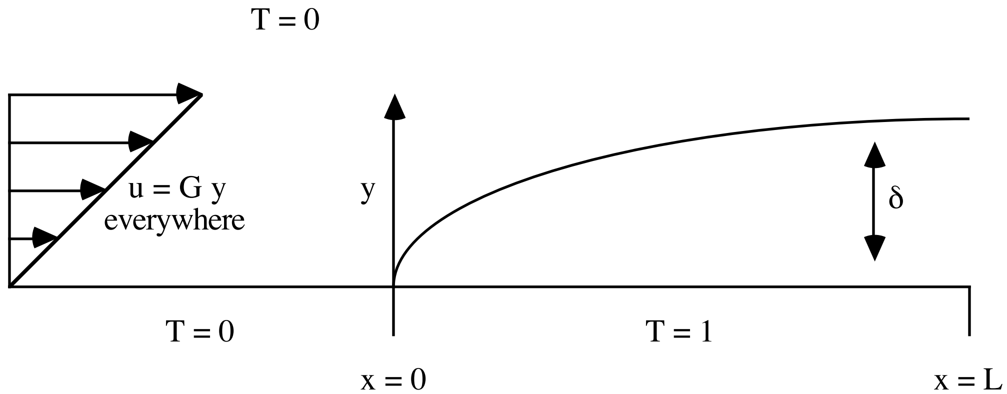
PROBLEM SET 11 CHEG 355 DUE 12/2/04

1. Reconsider the problem of a heated plate you looked at for homework last time, only now you will solve it!



$$G\, y \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$T\Big|_{y=0,\, x<0} = 0 \qquad T\Big|_{y>>0} = 0 \qquad T\Big|_{y=0,\, x>0} = 1 \qquad Q = \int_0^L -k \frac{\partial T}{\partial y}\, dx$$

a). By using the coordinate stretching technique illustrated in class, show that the boundary layer equation admits a similarity solution and obtain the similarity rule and similarity variable. Obtain the transformed ODE and boundary conditions. How does the thickness of the thermal boundary layer grow as it moves down the plate?

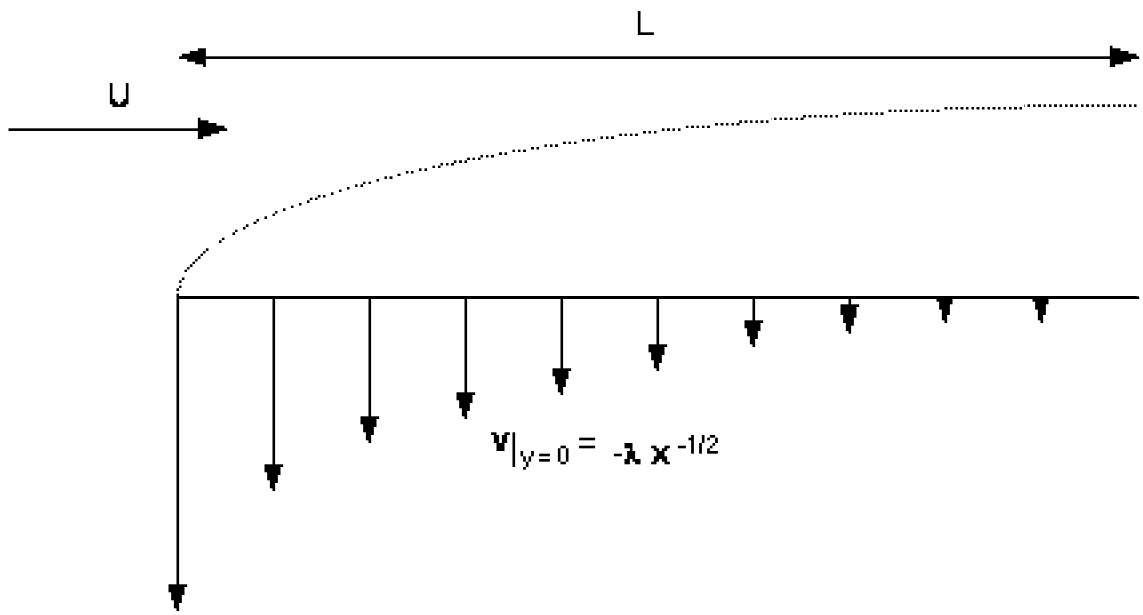
b). Solve the ODE. Note that f''/f' = (ln(f'))'. You may leave the final result in terms of an explicit integral of a known function, or you may evaluate the integral in terms of the incomplete gamma function (look it up in a handbook). Obtain a similar explicit relationship for the heat loss from the plate as a function of the length of the plate. Note that nearly all aspects of the solution except the final numerical value may be learned without explicitly solving the equation.

2. Boundary layer growth with suction: One technique used to control the rate of boundary layer growth on airplane wings is suction -- the wing (or plate) is porous, and fluid is sucked out of tiny holes which has the effect of keeping the boundary layer attached and preventing separation. In this problem we will examine the simple case of uniform flow past a flat plate where the vertical suction velocity is given by the power-law relation:

$$v\,\Big|_{y=0} = -\lambda\, x^{-1/2}$$

a. What should be the characteristic magnitude of λ to affect the boundary layer thickness (e.g., how should it scale with U, μ, ρ, L, etc.) and what should be the total amount of gas withdrawal (the integral of v over the plate)?

b. What dimensionless numerical value should λ^* (e.g., λ rendered dimensionless by the scaling determined in part a) take on to reduce the boundary layer thickness (e.g., the value of η where the velocity approaches half that of the free stream flow) by a factor of two? Note that this will require a numerical solution to the Blasius Equation - where your boundary condition $f(0) = 0$ is replaced by one which involves λ^*. The numerical part shouldn't take very long if you use the shooting method as you did last year (feel free to look up the solutions to problem 30 [4/20/01] of the old CHEG258 online course notes out of my directory). Plot up the boundary layer thickness and the wall shear stress (e.g., $f''(0)$) as a function of λ^*.



3. In class on Tuesday Trinidad, Claudia, and Pat demonstrated how to crush a can using the volume contraction of water vapor upon cooling and condensing. I would like you to do an analysis of this problem for a can of volume V and orifice A. If the can crushes if the pressure inside reaches half of atmospheric pressure, how fast do you have to cool the vapor in the can to get it to collapse? Do this using both variables, and then after measuring up a soda can, with actual numbers. Make any approximations necessary to get a tractable problem.