

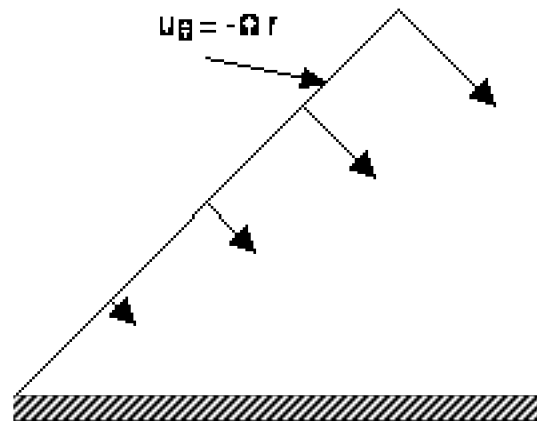
1. Flow inside a collapsing  $45^\circ$  wedge. Consider the flow depicted below. If we can ignore the inertial terms ( $Re \ll 1$ ), the flow is governed by the Biharmonic Equation. The velocity at  $\theta = 0$  is zero, while that at  $\theta = \theta_0 = \pi/4$  is  $u_r = 0$ ,  $u_\theta = -\Omega r$  where  $\Omega$  is the angular velocity of the upper plate.

a. Using the trial function employed for the wiper problem discussed in class, solve for the streamfunction.

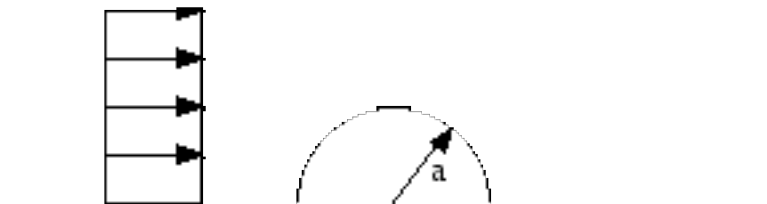
b. Using the matlab contour command, plot up the streamlines.

Hint: The streamfunction will be of the form  $\psi = 0.5 \Omega r^2 f(\theta)$ .

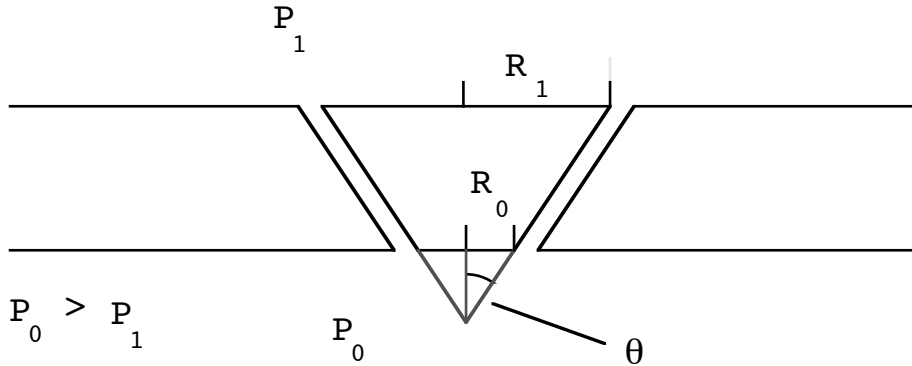
Second Hint: The four homogeneous solutions for  $f(\theta)$  are  $1$ ,  $\theta$ ,  $\sin(2\theta)$ , and  $\cos(2\theta)$ .



2. For ideal potential flow, calculate the forces on a cylinder imbedded in a plane as depicted below. You may assume that in the absence of the cylinder the flow is uniform in the  $x$ -direction. Determine whether the calculated force is physically realizable, and if not state why.



3. A conical cork is used to control the flow of air through a conical hole as is depicted below. For what values of  $R_1$  and  $R_0$  will the plug be blown out of the hole? The flow is considered to be ideal and inviscid, and the cork is massless. You should find that the force on the cork is independent of both the conical angle  $\theta$  and the width of the gap surrounding the cork.



Hint: Assuming parallel flow in the gap, use Bernoulli's equation and continuity to determine the velocity in the gap.

Double Hint: To simplify the algebra, use a coordinate system  $s$  defined from the imaginary vertex of the cone, calculate  $P(s)$ , and integrate over  $s$  from  $s_0$  to  $s_1$  where  $s_0 = R_0/\sin\theta$  and  $s_1 = R_1/\sin\theta$ .

4. Dimensional Analysis of Flow Down an Inclined Plane: Consider the inclined plane depicted below. The plane is inclined by an angle  $\theta$  from the horizontal, and a fluid (viscosity  $\mu$ , density  $\rho$ ) is flowing down the plane with flow rate  $Q/W$  per unit width of the plane (normal to the plane of the paper - this is a two-dimensional problem). We wish to determine the thickness of this fluid layer as a function of the parameters of the problem using dimensional analysis.

a. Form a dimensional matrix and prove that the problem involves just three dimensionless groups. Determine three independent dimensionless groups (Hint: one will be the Reynolds number).

b. At low Reynolds numbers we anticipate that the flow rate will be proportional to gravity. Use this to strengthen the result of the dimensional analysis, and determine the relationship between the flow rate and fluid thickness to within some unknown function of the angle of inclination. The thickness is proportional to what power of  $Q/W$ ?

c. Solve the problem.

