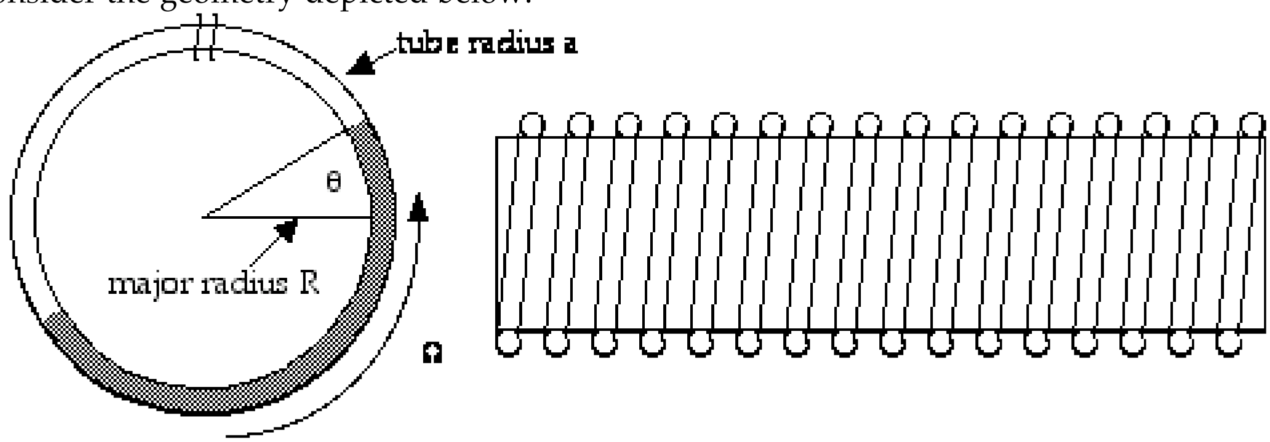
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Unidirectional Flows: Last week Lucas, Samantha, and Caroline demonstrated the Archimedes Screw, an ancient pumping technique. In this problem we explore the Coil Pump, a variant which is still used in irrigation systems in parts of the world. Consider the geometry depicted below:



A long tube of radius a is coiled around a horizontal drum of radius R (we take $a/R \ll 1$). As the drum rotates with angular velocity Ω, each loop of the coil is half-filled with fluid and air. Gravity drags the fluid down towards the bottom of each loop of the coil, and causes the fluid to spiral its way out to the exit. Because of frictional losses, however, the fluid slug in each loop is dragged up a bit, so that instead of being level it is at some angle θ from horizontal. Here we analyze the steady-state performance in the absence of any back pressure!

a. What is the nominal *liquid* flow rate of the pump as a function of the parameters of the problem? This is easy!

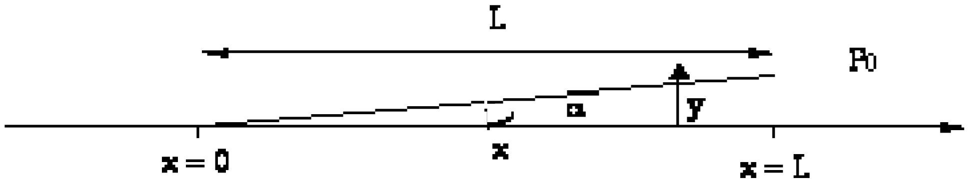
b. If the flow is laminar, we can approximate the frictional resistance as that due to unidirectional flow through a tube. Using this, determine the angle θ the slug in each loop acquires relative to horizontal as a function of the angular velocity. Work in a reference frame where the slug is stationary (as depicted above). The z-momentum equation may be helpful:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

c. The maximum possible flow rate occurs when θ reaches $\pi/2$. What is the maximum angular velocity and flow rate?

d. The key assumption leading to the answers in part b and c is laminar flow, which is valid only for Re < 2100 (based on tube diameter). If R = 10 cm, a = 1 cm, and the working fluid is water, calculate whether this assumption is correct. If the working fluid is corn oil ($\nu \sim 80$ centistokes) what will this answer be? Are we underestimating or overestimating the maximum flow rate?

Problem 2). (20 points) Lubrication: An important aspect of lubrication is *cavitation*, which occurs when the absolute pressure in a gap gets below the vapor pressure of the liquid (or an absolute pressure of zero, whichever comes first). Consider the geometry depicted below. A plate has one edge resting on a plane, such that the two surfaces are separated by an angle α, where $\alpha \ll 1$ (the lubrication limit). By pulling up on the right edge of the upper plate, it rotates with an angular velocity Ω. Your goal is to determine how close to the corner the fluid cavitates in the limit of small vapor pressures.



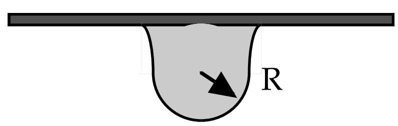
a. Write down the equations governing the velocity distribution in the gap and scale them in the lubrication limit.

b. Solving these equations, determine the position x_c where cavitation occurs.

Problem 3). (10 pts) Dimensional Analysis: Under some conditions (slow drips seeping from the underside of a porous plate) the radius R of a drop detaching from the surface is the result of a balance between the surface tension Γ (units Force/Length) and gravitational forces.

a. Form the dimensional matrix of the parameters governing this problem, and determine the number of dimensionless groups involved.

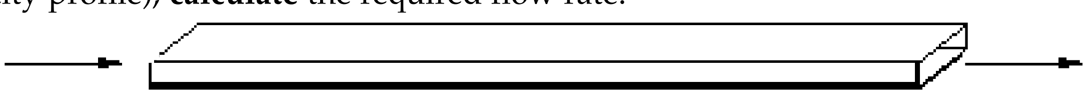
b. Using the results of part (a) give a **rough estimate** of the radius R of a drop of water ($\Gamma = 70$ dynes/cm) dripping off of a plate and calculate its volume.



Problem 4). (10 points) A *very* familiar problem (with different numbers) - do it right this time!: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is 100μm deep and 1mm wide in cross-section, and 2cm long. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 1mm x 2cm surface - where cell adhesion is being studied) of 10 dyne/cm^2. Due to the ratio of length scales, you can assume unidirectional plane-Poiseuille flow. We want to determine the required flow rate if the working fluid is water.

a. By scaling the boundary conditions, **estimate** the required flow rate.

b. Using the flow equations (or simply taking advantage of your knowledge of the shape of the velocity profile), **calculate** the required flow rate.



Problem 5). (20 points) Short Answer / Multiple Choice:

Identify the following equations and state under what conditions they are valid:

1). $\frac{\partial \sigma_{ij}}{\partial x_j} = 0$

2). $\nabla^4 \psi = 0$

Write out the term which corresponds to the following physical mechanisms:

3). Diffusion of x-momentum in the y-direction.

4). Convection of x-momentum in the y-direction.

5). The coriolis force, and which component of the NS equations (r, θ, or z) does it appear in?

6). If two identical spheres of radius a are settling due to the force of gravity at zero Reynolds number in an infinite fluid (no boundaries), how does the vector drawn between the sphere centers change with time? Justify your answer.

7). Order the drag of the following objects (from lowest to highest) under creeping flow conditions

A. A cube 2 cm on a side
B. A sphere 2 cm in diameter
C. A sphere 1.75 cm in radius

8). Estimate the Reynolds number associated with the swimming motion of my big orange goldfish. Make any approximations (or guesses, if you haven't seen him) necessary, but give the basis of your estimate. Is this motion dominated by viscous or inertial effects?

9). A paramecium is a single celled animal around 200μm in size that tends to swim around in circles with a velocity of 1mm/s. Estimate the Reynolds number associated with its motion. Is this motion dominated by viscous or inertial effects?

10). In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 10m boat of this type? Show your equation!