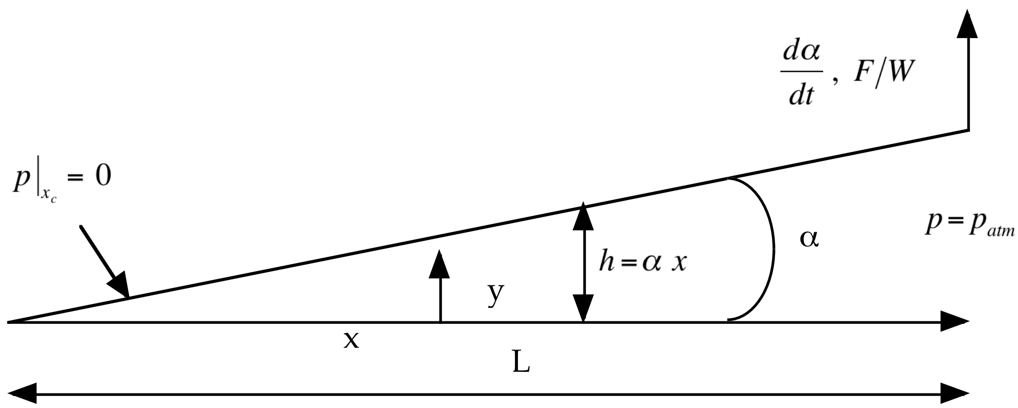
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Lubrication: Cavitation is the phenomenon which occurs when the absolute pressure in a liquid reaches the vapor pressure, and the liquid boils. For most liquids this is pretty close to zero. While usually seen in inertial flows, it is actually most easily visualized in viscous lubrication. Consider the geometry depicted below:



Two plates of length L in contact at one edge are separated by a very small angle α. For small angles the gap between the plates is given by h = α x, where x is the distance from the vertex. The plates are pulled apart at some rate dα/dt.

a. Using lubrication analysis, determine the location x_c where the pressure falls to zero absolute (e.g., a gauge pressure of - p_{atm}) and the fluid cavitates.

b. What is the force per unit width into the paper F/W (exerted at the outer edge) necessary to pry the plates apart for some angular velocity dα/dt? (Hint: Think of torque balances! You may leave this in terms of an integral if you wish.)

Problem 2. (20 pts) Inspectional Analysis: It is proposed to model the drag on a large ship by towing a 1:100 scale model in a towing tank. You are assigned the task of doing the modeling.

a. Write down the Navier-Stokes equations in vector form and render them dimensionless using the inertial scaling. Identify the two dimensionless groups which appear.

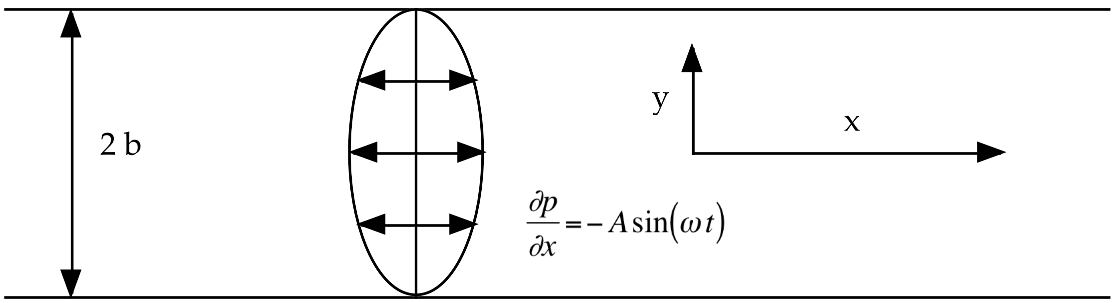
b. If the target velocity of the large ship is U, what should the velocity U_m of the scale model be? (Hint: what dimensionless group are you trying to preserve?)

c. You want to size the power plant required to drive the ship. If the force measured on the towing rope for the model is F_m, what is the required size of the power plant? (As a practical matter, you would also need to account for losses, so the required size would be greater than this value – ignore this effect.)

Problem 3). (20 points) Unidirectional Flows: Consider the **unsteady** oscillatory flow in a channel of width 2 b depicted below. The fluid is incompressible, and the flow is unidirectional in the x-direction, with all that implies (**Hint**: Remember which of the inertial terms vanish!). We impose an oscillatory pressure gradient given by:

$$\frac{\partial p}{\partial x} = -A\sin(\omega t)$$

where A is the gradient amplitude and ω is the frequency of oscillation in time (the fluid sloshes back and forth in the x-direction).



a. Write down the momentum equation in the x-direction and show which terms are zero. Render the equations dimensionless using $t^* = \omega t$ as the dimensionless time and U_C as an unknown velocity scale. Divide out by A to make the equations dimensionless. The characteristic velocity U_C is determined by balancing the driving force in the problem (the pressure gradient) with either the inertial or viscous term. Recognizing this, determine this characteristic velocity for 1) high, and 2) low frequencies, and explicitly identify the single dimensionless group the problem depends on.

b. Solve for the velocity profile in the high frequency limit. What happens to the boundary conditions?

c. Solve for the velocity profile in the low frequency limit.

d. For this sort of problem, we are primarily interested in the amplitude of the displacement of the fluid in the x direction, which is easily obtained by integrating the velocity over one half period in time. Using the results of b and c, determine the amplitude Δx at the centerline of the flow in both the high and low frequency limits.

Problem 4). (20 points) Short Answer / Multiple Choice:

a. (10 pts) Briefly identify the physical mechanism (or equation name) described by each of the following terms **and a problem where it would play a role**:

1. $\mu \dfrac{\partial^2 v_\theta}{\partial z^2}$

2. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

3. $\nabla^4 \psi = 0$

4. $\rho \dfrac{v_r v_\theta}{r}$

5. $p + \frac{1}{2}\rho u^2 + \rho g h = Cst$

b. (10 pts) Multiple Choice:

1). Which (if any) of the following can be discontinuous at a fluid-fluid interface?
- A. Shear stress
- B. Heat flux
- C. Mass flux
- D. Velocity

2). Order the drag of the following objects under creeping flow (Re << 1) conditions
- A. A cube 2 cm on a side
- B. A sphere 1.75 cm in radius
- C. A sphere 2 cm in diameter

3). 2-D Stokes flow has a simple analogy in solid mechanics. What is it, and how does it work? No more than two sentences!

4). A paramecium is a single celled animal around 200μm in size that tends to swim around in circles with a velocity of 1mm/s. Estimate its Reynolds number.

5). In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 10m boat of this type?