PROBLEM SET 1 CBE30355 DUE 8/28/12

These first few problems should serve as a review for the vector calculus material you learned last semester and will use extensively this term.

1). Calculate the angle between the following pairs of vectors ($\mathbb{e}_x$, $\mathbb{e}_y$, $\mathbb{e}_z$):

a. (1,0,1), (1,0,0)

b. (1,0,1), (0,1,0)

c. (1,2,3), (3,2,1)

2). Calculate the following quantities (Note: X denotes the cross-product and · the dot product):

a. (1,0,1)·(1,0,0)

b. (1,0,1)·(0,1,0)

c. (1,2,3)X(3,2,1)

3). For the scalar potential function $\phi = (x^2+y^2+z^2)^2$ and the velocity vector field $\mathbb{u} = (y^2, z, x^2)$ calculate the following vector quantities:

a. $\mathbb{\nabla} \phi$; $\mathbb{\nabla} \cdot \mathbb{u}$

b. $\nabla^2\phi = (\mathbb{\nabla} \cdot \mathbb{\nabla})\phi$; $\nabla^2 \mathbb{u}$

c. $\mathbb{\nabla} X \mathbb{u}$

where the boldface operator $\mathbb{\nabla} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

4. Prove that for an arbitrary scalar function ϕ:

$$\mathbb{\nabla} X(\mathbb{\nabla} \phi) = 0$$

5. This is <u>completely</u> optional (and not for credit - solve only if you like puzzles): Prove the following vector identity for the arbitrary vector $\mathbb{u}$:

$$\mathbb{\nabla} \; X (\mathbb{\nabla} \; X \mathbb{u}) = \mathbb{\nabla} \; (\mathbb{\nabla} \cdot \mathbb{u}) - \nabla^2 \mathbb{u}$$

Hint: I usually solve this problem using index notation which is very useful for describing advanced transport problems. Detailed notes on index notation are available through the class website. We'll go over index notation in the first few review sessions.

In this problem set all vectors are in outlined boldface type while scalars are in regular type.

6. A practical thought problem: Your friend fills his (or her) cup with diet soda at the LaFortune Burger King, takes a sip, and says “that tastes funny – I wonder if it’s really regular?” Come up with a way to definitively determine whether the drink is really diet or not. The testing procedure shouldn’t take more than a minute or two (e.g., you can’t wait for a puddle of the stuff to dry out to see if it is sticky) and should just use the resources available at the drink station.

As a side note, last Spring this actually happened to me. I was wondering why I was having trouble controlling my weight: it turned out that they had cross-connected the syrups and I was sucking down an extra 850 calories that I hadn’t accounted for every time I ate at BK. Pretty aggravating for me (it added up to an extra pound or two of fat, which took about 100 miles of biking to burn off), but it could be potentially serious for diabetics who have to watch their sugar intake.