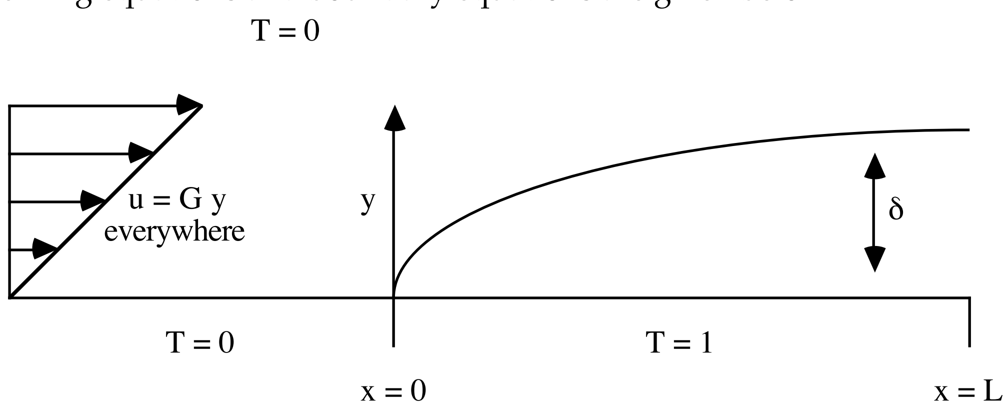
PROBLEM SET 11 CBE30355 DUE 11/27/12

1. Last week you scaled the problem of shear flow past a heated plate. This time you will solve it!

a). By using the coordinate stretching technique illustrated in class, show that the boundary layer equation admits a similarity solution and obtain the similarity rule and similarity variable. Obtain the transformed ODE and boundary conditions. How does the thickness of the thermal boundary layer grow as it moves down the plate?

b). Solve the ODE. Note that f''/f' = (ln(f'))'. You may leave the final result in terms of an explicit integral of a known function, or you may evaluate the integral in terms of the incomplete gamma function (you can look it up in a handbook, or online). Obtain a similar explicit relationship for the heat loss from the plate as a function of the length of the plate. Note that nearly all aspects of the solution except the final numerical value may be learned without explicitly solving the equation.

Recall that the problem was flow past the flat plate y=0 as depicted below. Fluid with a dimensionless temperature T = 0 flows along the plane in the x direction with velocity u = Gy where G is the shear rate (e.g., plane Couette flow without the upper plane). The plate is maintained at a temperature T = 0 for x < 0 and a temperature T = 1 for x > 0. The governing equations and boundary equations are given below.



$$G\,y\,\frac{\partial T}{\partial x} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

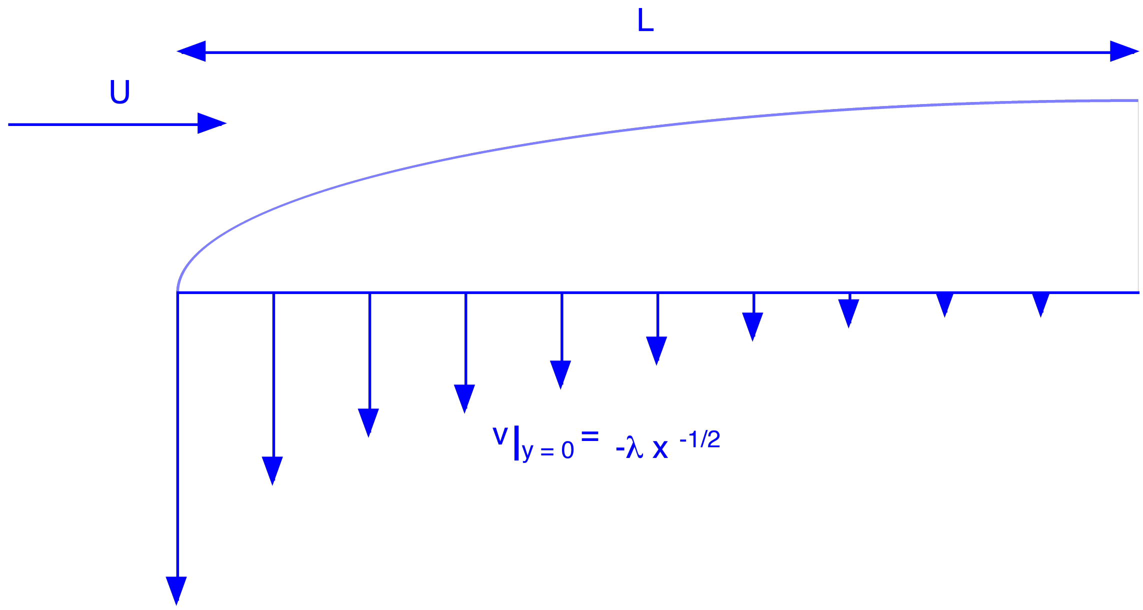
$$T\Big|_{y=0,\,x<0} = 0 \qquad T\Big|_{y>>0} = 0 \qquad T\Big|_{y=0,\,x>0} = 1 \qquad Q = \int_0^L -k\,\frac{\partial T}{\partial y}\,dx$$

2. Boundary layer growth with suction: One technique used to control the rate of boundary layer growth on airplane wings is suction -- the wing (or plate) is porous, and fluid is sucked out of tiny holes which has the effect of keeping the boundary layer attached and preventing separation. In this problem we will examine the simple case of uniform flow past a flat plate where the vertical suction velocity is given by the power-law relation:

$$v|_{y=0} = -\lambda x^{-1/2}$$

a. What should be the characteristic magnitude of λ to affect the boundary layer thickness (e.g., how should it scale with U, μ, ρ, L, etc.) and what should be the total amount of gas withdrawal (the integral of v over the plate)?

b. What dimensionless numerical value should λ^* (e.g., λ rendered dimensionless by the scaling determined in part a) take on to reduce the displacement thickness by a factor of two? Note that this will require a numerical solution to the Blasius Equation - where your boundary condition $f(0) = 0$ is replaced by one which involves λ^*. The numerical part shouldn't take very long if you use the shooting method described in class (feel free to look up the solutions to problem 30 [assigned 4/20/01] of the old CHEG258 online course notes out of my directory). Plot up the boundary layer thickness and the wall shear stress (e.g., $f''(0)$) as a function of λ^*.



3. A popular student demonstration is the spin-up and spin down of a cylinder full of water (the tea leaf problem). It was shown that boundary effects dominated the time to reach steady state, as it occurred much faster than would be expected from diffusion of momentum in from the side walls. In this problem, we will use scaling analysis to estimate the boundary layer thickness of a spinning disk analogous to the rotating bottom of the cylinder in the demonstration. Consider a disk of radius R spinning with angular velocity Ω in an infinite fluid at rest. By scaling the r, θ and z momentum equations, estimate the boundary layer thickness and radial velocity as a function of the parameters in the problem. What dimensionless group must be small in order for the boundary layer scalings to apply?