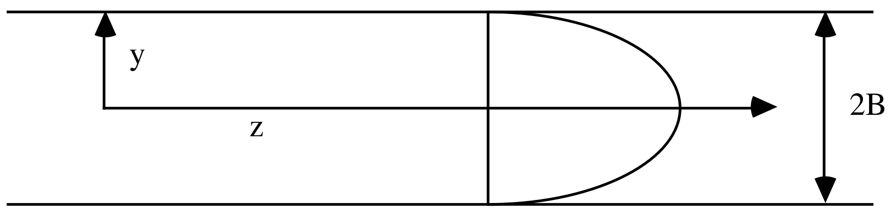
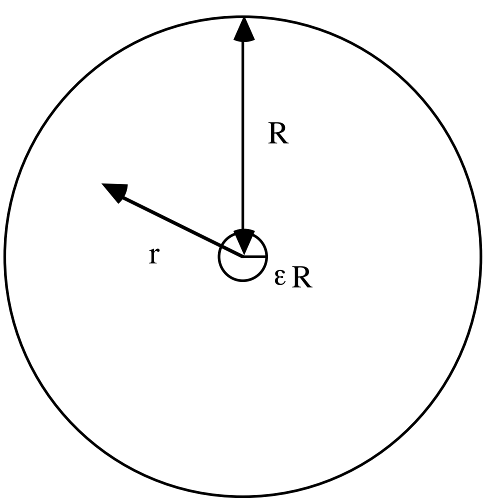
Read through Chapter 3 of BS&L.

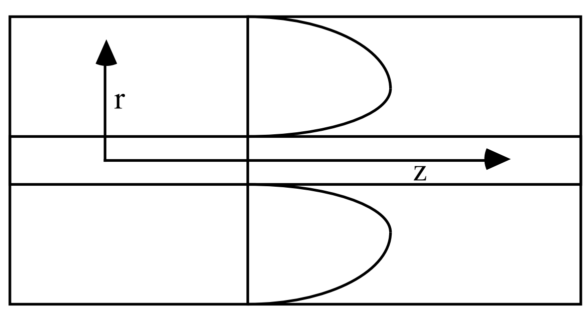
Remember that for perverse reasons BS&L defined the viscous stress tensor τ_{ij} to have a sign opposite of that defined in class and used in most other textbooks. If you keep this in mind, and recall that the sign of the stress tensor is completely arbitrary (as long as you are consistent!), it should not cause you any confusion.

1). Consider a viscous fluid flowing in a laminar manner through a slit formed by two parallel walls a distance 2B apart as is depicted below. Given that we have a uniform pressure gradient in the negative z direction (i.e., dp/dz equals a negative constant) and the fluid is massless with viscosity μ, calculate the flow rate using the Navier-Stokes equations. How does this answer change if the fluid has density ρ and gravity is in the positive z direction? Also, for both parts, calculate the force/area exerted by the fluid on the walls.

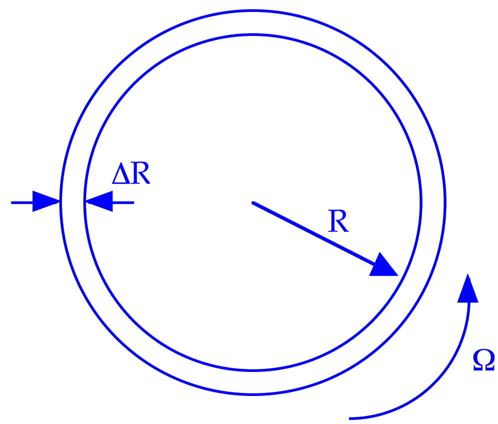


2. In class on Tuesday we will discuss the flow of a fluid through a pipe driven by a pressure gradient, as in problem 1 above. In this problem, consider a pipe of radius R with a small cylindrical wire of radius εR running axially down the center. If the remaining space in the tube is filled with a viscous fluid and a uniform pressure gradient is applied in the negative z direction, calculate the resulting flow rate (assume that gravity is unimportant) as a function of ε and compare its magnitude to that when the wire is absent. The velocity profile needs to be obtained analytically, and isn't too bad, but the flow rate gets a bit messy. You may do the flow rate numerically (plotting it up as a function of ε) if you wish. By how much is the flow rate reduced if $\varepsilon = 0.1$? Is this surprising? Also, determine the force per unit length exerted by the fluid on the wire.





3. In class we derived a way to estimate the viscosity of a fluid using a Couette viscometer by neglecting the curvature effects (e.g., if the gap width is ΔR and the inner radius is R, we required $\Delta R/R << 1$). Using cylindrical coordinates, derive the exact relationship between torque and rotation rate for arbitrary $\Delta R/R$ and graphically compare (e.g., use matlab to plot them up) the two results for $\Delta R/R$ in the range $0 < \Delta R/R < 1$. How large is the error in the approximate formula if $\Delta R/R = 0.1$?



4. In the last homework you determined the complete *mobility tensor* for a falling body of revolution in a viscous fluid from two simple experiments: the measured velocity point on, and the measured velocity broadside on. In this problem I want you to use this result to determine the trajectory of the rod as a function of p_i. If the director p_i is given by:

$$p_i = \delta_{i3} \cos(\theta) + \delta_{i1} \sin(\theta)$$

and the two measured velocities are 0.3 cm/s and 0.1 cm/s, for what angle θ is the lateral displacement the largest (e.g., maximum u_1/u_3)? We take gravity (and hence the force) to be in the δ_{i3} direction.