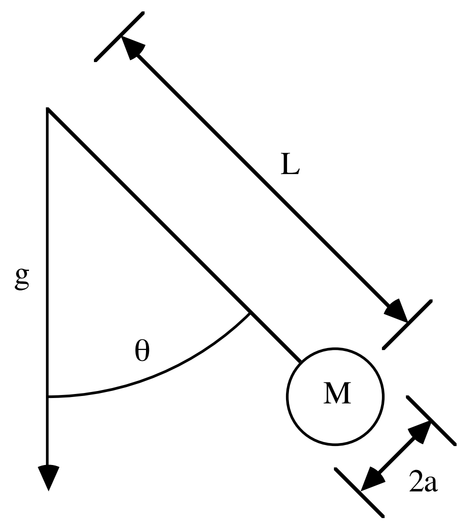
PROBLEM SET 7 CBE 30355 DUE 10/25/12

1. Dimensional analysis in cooking a turkey: If you've ever looked at the cooking timetables for large things like turkeys or roasts, you may have noticed that they are pretty complex: so much per pound if the weight is in one range, and different length in another range, etc. In general, the cooking time per pound goes down the larger the bird. If we assume that 1) all turkeys are geometrically similar, 2) the physical properties such as thermal diffusivity, density, etc., are all constant, and 3) the cooking time depends only on the mass, the density, and the thermal diffusivity (for idealized turkeys, anyway), use dimensional analysis to show how cooking time varies with the mass of the bird. Graphically compare your model (determining the constant empirically) to the values obtained from any source you wish. (If you don't have a cookbook, a web search on "turkey cooking time" is quite useful!)

2. Consider the damped pendulum depicted below. The pendulum consists of a ball of mass m and radius a suspended by a fine wire of length L below a pivot. The ball is moving through a fluid of viscosity μ (the fluid is assumed massless for convenience) which retards its motion. It is assumed that the ball moves sufficiently slowly that Stokes Law applies. Under these conditions, the equation describing the motion of the sphere is given by:



$$M L \frac{d^2\theta}{dt^2} = - M g \sin(\theta) - 6\pi \mu a L \frac{d\theta}{dt}$$

$$\theta_{t=0} = \theta_0 \ ; \ \left.\frac{d\theta}{dt}\right|_{t=0} = 0$$

a. By choosing an appropriate reference time scale, render the equation and initial condition dimensionless. Show that the dimensionless solution depends on only two dimensionless groups. What are the physical meanings of these parameters?

b. You are assigned the problem of adjusting the fluid viscosity so as to bring the sphere to rest as rapidly as possible. Recognizing that if $\mu = 0$ the pendulum will oscillate forever, and if μ is very large the sphere will take a long time to get to the equilibrium ($\theta = 0$) position, estimate the correct value of the viscosity for <u>critical</u> damping. Estimate from dimensional analysis how long it will take the sphere to approach to within, say, 25% of the equilibrium position.

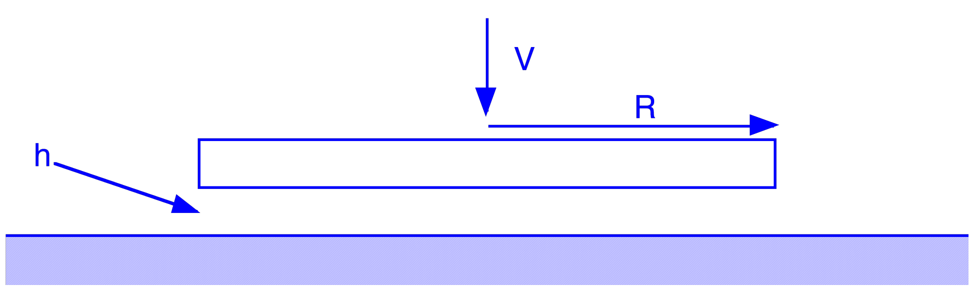
c. Given that θ_o is very small, we may make the approximation $\sin(\theta) = \theta$. Solve the resulting differential equation, and compare the exact solution to that estimated by dimensional analysis above.

3. In a recent paper (Davies & Stokes, J. Rheol 2005) a fundamental problem with current parallel-plate viscometers was identified. As you know, the viscosity of a fluid is measured in a parallel-plate viscometer by rotating one plate at a known angular velocity and measuring the resultant torque on the other plate. The viscosity calculation, however, requires accurate knowledge of the gap width between the plates. In a modern instrument it is possible to set the gap accurately automatically -provided- you have the reference point of where the two plates are in contact: e.g., where the gap is zero. For an instrument such as this, you just push a button, the upper plate decends at a constant velocity of $50\mu m/s$, and the gap is considered to be zero when the measured upward thrust on the upper plate exceeds 10^4 dynes, providing the reference point for all subsequent gaps - very convenient! This would be fine if there were no fluid in the gap (e.g., a vacuum), but as you know the viscosity of air is non-zero (1.8×10^{-4} poise). In this problem we analyze what the effect of air is on the gap zeroing of 3cm radius parallel-plates.

a. For a normal force detection threshold F, approach velocity V, air viscosity μ, and plate radius R, calculate the error in the gap set zero.

b. For the numbers given above, calculate the numerical value of the error in microns. Is it practical to reduce this error by an order of magnitude by decreasing V? Be specific!

c. If we set the gap to $200\mu m$ using this incorrect zero (e.g., actual gap differs from $200\mu m$ due to the zero error), what is the corresponding error in the viscosity measurement μ_{exp}/μ of a fluid?



4. By taking the divergence of the equations of motion and applying the equation of continuity, prove that $\nabla^2 p = 0$ for an incompressible fluid undergoing flow at zero Reynolds number. Use index notation only, and note that the ∇ and ∇^2 operators commute.