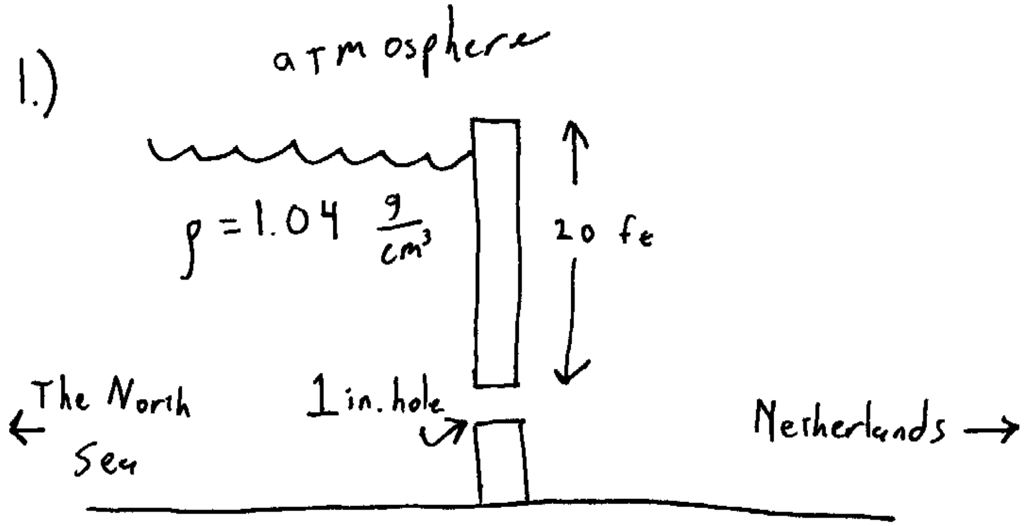
CBE 30355 Problem Set 2 $\boxed{\text{Solution}}$

1.)



a.) Find the force necessary to plug a 1 inch hole.

$$F_{plug} = (\text{pressure @ 20 ft})(\text{Area})$$

$$F_{plug} = P_w A$$

Note: atmospheric pressure is acting on both sides of the dike, so it adds nothing to F_{plug}.

Let's use SI units (MKSA)

$$A = \left(\frac{1 \text{ inch}}{2}\right)^2 \pi = r^2 \pi$$

$$A = .25\pi \text{ inches}^2 \left[\frac{1 \text{ m}^2}{1550 \text{ in}^2}\right] = 6.45 \times 10^{-4} \frac{m^2}{\cancel{in^2}}(\pi)(.25)\cancel{in^2}$$

$$= 5.067 \times 10^{-4} m^2$$

$$P_w = \rho_w g h \doteq \left(1040 \frac{Kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(20 ft)\left[\frac{1 m}{3.28 ft}\right]$$

$$= 62,100 \frac{Kg}{m \cdot s^2} = 62.1 \text{ kPa} \approx 9 \text{ psi}$$

$$F = P_w A = \left(62,100 \frac{Kg}{\cancel{m} \cdot s^2}\right)\left(5.07 \times 10^{-4} m^{\cancel{2}}\right) = 31.5 \frac{Kg\, m}{s^2}$$

$$\boxed{F_{plug} = 31.5 \text{ N}} = 3.15 \times 10^6 \text{ dy}$$

$$= 7.1 \text{ lb}_f$$

1.) continued

b.) $A = \left(\frac{12 \text{ inch}}{2}\right)^2 \pi = 36\pi \text{ inches}^2 \left[\frac{1 m^2}{1550 \text{ in}^2}\right] = 0.073 \, m^2$

$P_w = 62{,}100 \, \frac{Kg}{m \cdot s^2}$ (assume that the pressure is constant over the area of the hole - which it is, <u>on</u> <u>average</u>)

$F_{plug} = P_w A$

$F_{plug} = \left(62{,}100 \, \frac{Kg}{m \cdot s^2}\right)\left(0.073 \, m^2\right)$

$\boxed{F_{plug} = 4533 \, N}$

$= 4.5 \times 10^8$ dyne

$= 1{,}019 \, lb_f$

2.)

$\longrightarrow$

2. Viscosity.

From Perry's,

μ air = 1812×10^{-7} p = ~~0.01806 cp~~ 0.01812

μ O_2 = 2026×10^{-7} p = 0.02026 cp

μ N_2 = 1766×10^{-7} p = 0.01766 cp

μ CO_2 = 1463×10^{-7} p = 0.01463 cp

From Bird, Stewart, & Lightfoot:

$$\mu\ (Pa \cdot s) = 2.67 \times 10^{-6} \frac{(MT^{\theta})^{1/2}}{\sigma \Omega_v} = 2.67 \times 10^{-3} \frac{(MT^{\theta})^{1/2}}{\sigma \Omega_v} c$$

Estimate Ω_v from Fig. 3.4.2

M = molecular weight

σ =

T = dimensionless temp

K/ϵ = 1/critical temp.

Table: (for T = 293.15 K = 20°C)

gas:	M	σ	K/ϵ	T^0	Ω_v	μ calc (cp)	μ actual (cp)
air	29	3.71	1/78.6	3.73	0.97	0.01843	0.01812
CO_2	44	3.94	1/195	1.50	1.25	0.01563	0.01463
N_2	28	3.8	1/71.4	4.11	0.94	0.01782	0.01766
O_2	32	3.47	1/107	2.74	1.1	0.01952	0.02026

Answer →

2) continued

Answers

Gas	% difference
air	1.71 %
CO_2	6.8 %
N_2	0.9 %
O_2	3.7 %

$$= \frac{|\mu_{calc} - \mu_{actual}|}{\mu_{actual}}$$

3.) Viscosity relationship to temp.

exponential:

$$\mu = A \exp\left[B/T \right]$$

To find the constants A & B, should linearize:

$$\ln \mu = \ln A + B\left(\frac{1}{T}\right)$$

Plot $\ln \mu$ vs. $\frac{1}{T(K)}$ for glycerine.

Fit a linear trendline via least-squares:

$$\ln(\mu_{glycerine}) = -15.75 + 6,771\left(\frac{1}{T}\right)$$

Constants: $\boxed{\begin{array}{l} A = -15.75 \\ B = 6,771 \end{array}}$ factor: $\frac{\mu_{21°C}}{\mu_{20°C}} =$ 0.924

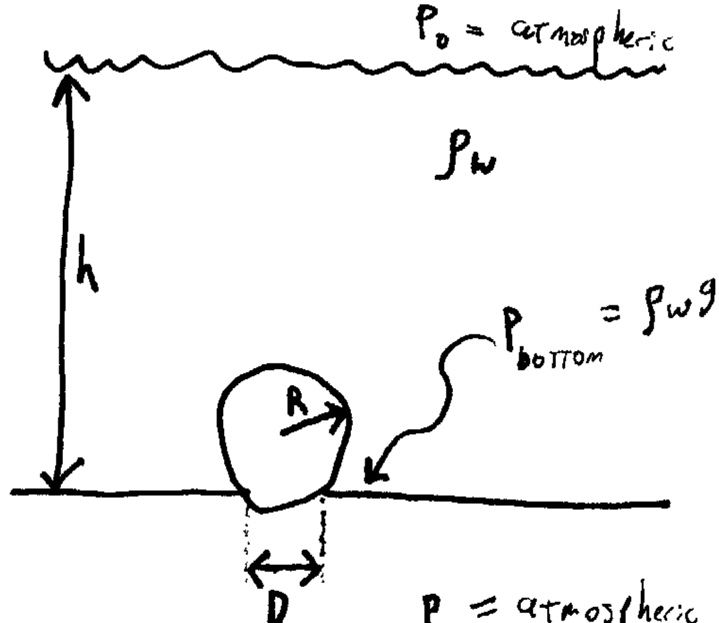
4.) Pool drain:

There are three ways of doing this problem.

1) quick & dirty
2) quick & clean
3) long & arduous

First way: Quick & dirty

We know that there are two forces in balance to keep the ball in place. There is the upward buoyancy force ↑, and the downward weight of the column of water above the drain ↓. Another way to think about this last force is that the pressure at the bottom of the pool is much greater than the pressure in the drain, so the ball is being "sucked" downward.



$$F_b + F_w = 0$$

F_b = buoyancy

F_w = weight of water = pressure difference

→

4) continued:

Let's look at F_b. If $R >> D$, then almost all of the ball is submerged. Then,

$$F_{buoyancy} = \rho_w g V_{ball}$$
$$= \frac{4}{3} \pi \rho_w g R^3$$

Now for F_w. the pressure difference at the drain is:

$$\Delta P = P_{bottom} - P_0$$
$$= (P_0 + \rho_w g h) - P_0$$
$$= \rho_w g h.$$

Multiply by the area of the drain to get the "sucking" force:

$$F_w = \frac{D^2}{4} \pi \rho_w g h$$

Lastly:

$$F_w + F_B = 0$$

$$\frac{D^2}{4} \pi \rho_w g h + \frac{4}{3} \pi \rho_w g R^3 = 0$$

Solve for $h = \frac{16}{3} \frac{R^3}{D^2}$

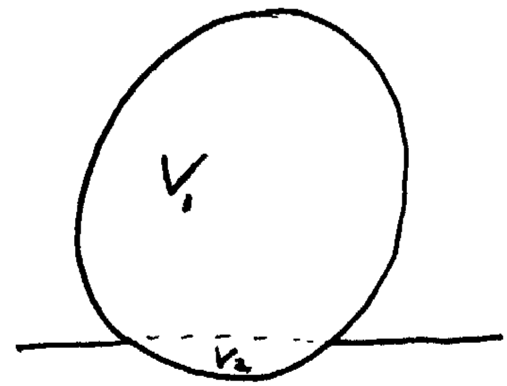
$$\boxed{h = 21.33 ft}$$

4.) <u>Continued:</u>

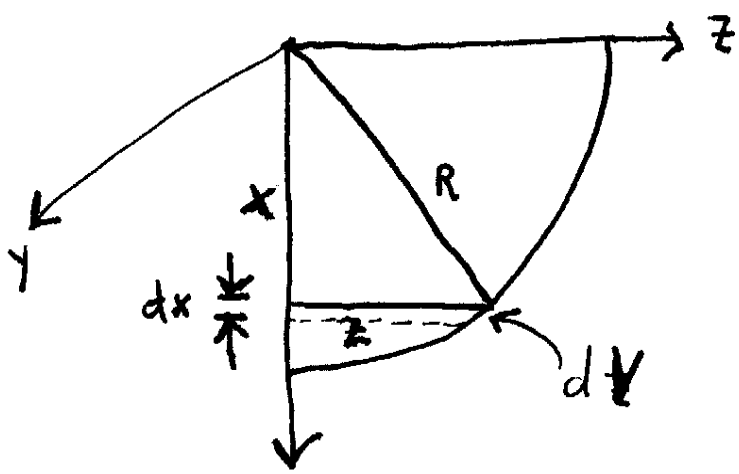
Second way: Quick & clean

Take another look at F_b. The displaced volume isn't quite the whole volume of the ball, but rather just the volume <u>above</u> the level of the drain hole, or V_1.

We could get V_1 if we knew V_2, since $V_{ball} = V_1 + V_2$.



To get V_2, consider the cross section below the drain:



- We want the differential volume dV occupied by dx.

$$dV = \pi z^2 \, dx.$$

- Pythagorus tells us that:

$$z = \sqrt{R^2 - x^2}, \quad \text{so}$$

$$dV = \pi\left[(R^2 - x^2)^{1/2}\right]^2 dx$$

Integrate from $x = x$ to $x = R$:

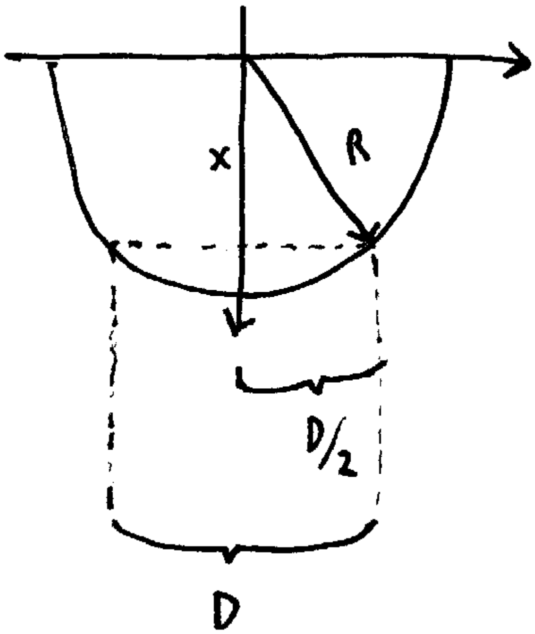
$$\int_x^R dV = \int_x^R \pi (R^2 - x^2) \, dx$$

$$V_2 = \left[\pi R^2 x - \pi \frac{x^3}{3}\right]_x^R = \frac{2}{3}\pi R^3 - \left(\pi R^2 x - \pi \frac{x^3}{3}\right)$$

$\longrightarrow$

4.) continued:

Now get x in terms of R and D:



$$x = \sqrt{R^2 - \frac{D^2}{4}}$$

So:

$$V_2 = \frac{2}{3}\pi R^3 - \left[\pi R^2\left(R^2 - \frac{D^2}{4}\right)^{1/2} - \frac{\pi}{3}\left(R^2 - \frac{D^2}{4}\right)^{3/2}\right]$$

$$V_2 = 0.003134 \text{ ft}^3$$

$$V_{ball} = 4.18879 \text{ ft}^3$$

$$V_1 = 4.18566 \text{ ft}^3$$

$$F_B = \rho_w g V_1 = \sim$$

$$F_B - F_w = 0$$

$$\rho_w g V_1 - \pi \frac{D^2}{4} \rho_w g h = 0$$

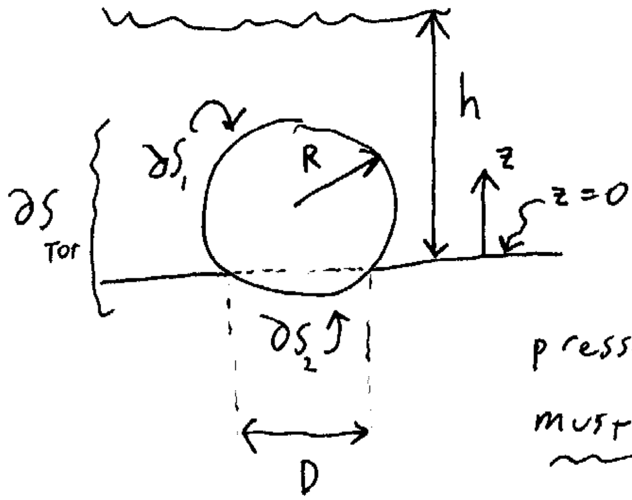
$$h = \frac{4V_1}{\pi D^2}$$

$$\boxed{h = 21.317 \text{ ft}}$$

- This answer is accurate, but it uses a lot of physical insight. If you want to do the problem without a priori knowledge, proceed to long & arduous

4. Continued

3rd way: long & arduous



We know that the pressure exerted by the fluid acts normal to the surface of ball. If we sum up the forces associated with that pressure over the entire surface, they must sum to zero for the ball to remain stationary.

$$F_z = \hat{e}_z \cdot \int_{\partial S_{tot}} -P \, \underline{n} \, dA = 0$$

(remember we're only interested in the z-direction)

$$= \int_{\partial S_{tot}} -P \, \underline{n} \cdot \underline{e}_z \, dA$$

$P = P_0 + \rho_w g (h - z)$ for $z > 0$

$P = P_0$ for $z < 0$.

$$= \int_{\partial S_1} -(P_0 + \rho_w g (h - z)) \, \underline{n} \cdot \hat{\underline{e}}_z \, dA + \int_{\partial S_2} -P_0 \, \underline{n} \cdot \hat{\underline{e}}_z \, dA$$

remember that the integral of a constant over a closed surface $= 0$, so we can drop the P_0.

$$0 = \int_{\partial S_1} -\rho_w g (h - z) \, \underline{n} \cdot \hat{\underline{e}}_z \, dA$$

~~[illegible]~~

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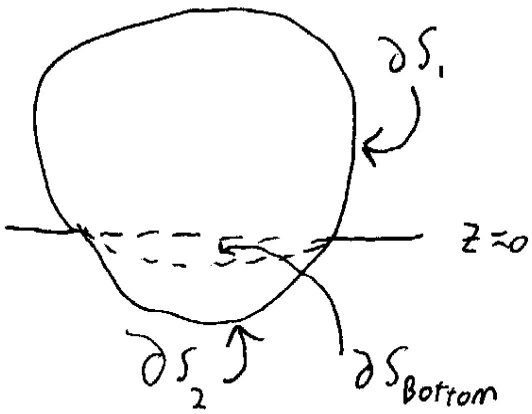
4. <u>continued</u>

Now, create a new surface ∂S_{Bottom}, which is in the plane of the bottom of the pool separating ∂S_1 and ∂S_2.



If we add and subtract the force = (pressure)·(area) integral over ∂S_{Bottom}, we can create a new closed integral surface.

$$0 = \int_{\partial S_1} -\rho_w g(h-z)\, \underline{n}\cdot\underline{\hat{e}}_z \, dA + \underbrace{\int_{\partial S_{Bot}} -\rho_w g \underbrace{(h-z)}_{h,\ \text{since } z=0} \underbrace{\underline{n}\cdot\underline{\hat{e}}_z}_{-1,\ \text{since } \underline{n}=-\underline{\hat{e}}_z \text{ for } \partial S_{Bot}} dA}_{-\rho_w g h \pi \frac{D^2}{4}} - \int_{\partial S_{Bot}} -\rho_w g(h-z)\, \underline{n}\cdot\underline{\hat{e}}_z$$

Combine these:

$$0 = \int_{\partial S_1 + \partial S_{Bot}} -\rho_w g(h-z)\, \underline{n}\cdot\underline{\hat{e}}_z \, dA \quad - \rho_w g h \pi \frac{D^2}{4} \quad \longrightarrow$$

4. Now apply the divergence theorem. Recall:

$$\iiint_V (\underline{\nabla} \cdot \underline{F})\, dV = \iint_{\partial V} \underline{F} \cdot \underline{n}\, dS \quad \text{for a closed volume } V.$$

Then:

$$-\int_{\partial S_1 + \partial S_{Bot}} -\rho_w g (h-z)\, \underline{\hat{e}_z} \cdot \underline{n}\, dA =$$

$$= \int_{V_1} -\rho_w g \big[\underbrace{\underline{\nabla}(h-z)}_{-\underline{\hat{e}_z}}\big] \cdot \underline{\hat{e}_z}\, dV - \rho_w g h \frac{\pi D^2}{4}$$

$$0 = \int_{V_1} \rho_w g \underbrace{\underline{\hat{e}_z} \cdot \underline{\hat{e}_z}}_{1}\, dV - \rho_w g h \pi \frac{D^2}{4}$$

$$0 = \rho_w g V_1 - \rho_w g h \pi \frac{D^2}{4}$$

Now we're back to quick & clean. So you get the same answer, but with a lot more work. It pays to know what the answer is before you start.

$$\boxed{h = 21.317 \text{ ft}}$$

5.) Find the pressure at the center of the earth

Assume $\rho = 5.67 \frac{g}{cm^3} = 5670 \frac{Kg}{m^3}$

$R_o = 3957$ miles $= 6.370 \times 10^6$ m

Recall: From physics, we know that $g_o = 9.8 \frac{m}{s^2}$ at the earth's surface, $g = 0$ at $r = 0$, and varies linearly in between, so:

$$g = \frac{r}{R_o} g_o$$

Also: From hydrostatics, $P = \rho g h$, or

$$\frac{dP}{dr} = \rho g(r).$$

Combining: $$\int_0^{R_o} dP = \int_0^{R_o} \rho \frac{g_o}{R_o} r \, dr$$

$$P_o = \frac{\rho g_o R_o^{\cancel{2}}}{2 \cancel{R_o}} = \frac{\left(5.67 \times 10^3 \frac{Kg}{m^3}\right)\left(9.8 \frac{m}{sec^2}\right)\left(6.37 \times 10^6 m\right)}{2}$$

$$P_o = 1.77 \times 10^{11} \ Pa$$

$$\boxed{P \simeq 1.75 \times 10^6 \ atm}$$