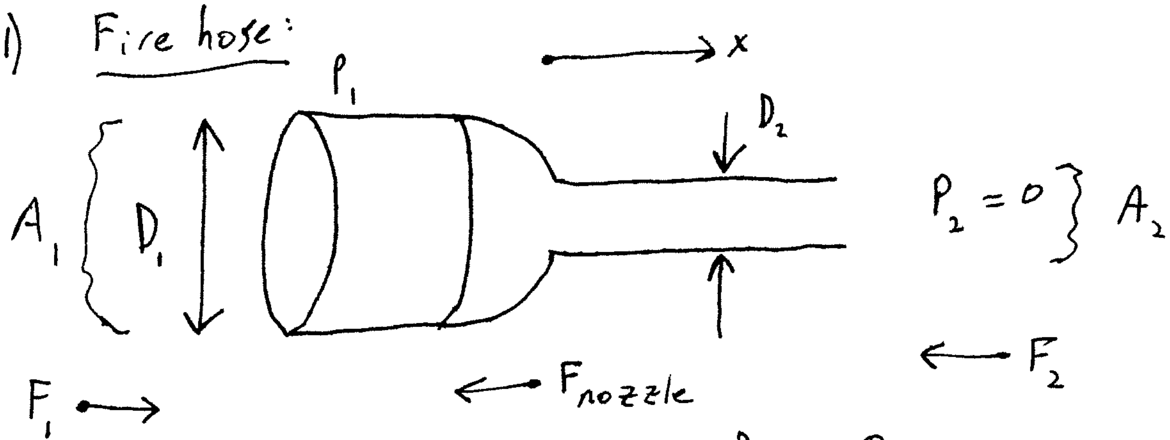
1) Fire hose:



$$P_1 = 30 \text{ psig} \qquad P_2 = 0 \text{ psig}$$

$$D_1 = 4'' \qquad D_2 = 2''$$

$$Q = 40 \text{ gal/sec } H_2O$$

$$\sum F_x = \int_{\partial D} \rho \underline{u} (\underline{u} \cdot \underline{n}) \, dA \cdot \underline{e}_x \qquad (i)$$

$$\sum F_x = (F_1)_x + (F_2)_x + (F_{nozzle})_x \qquad (LHS)$$

$$= P_1 A_1 - P_2 A_2 + (F_{nozzle})_x \qquad (ii)$$

Combine equations (i) and (ii), solve for $(F_{nozzle})_x$:

$$(F_{nozzle})_x = P_2 A_2 - P_1 A_1 + \int_{\partial A_{nozzle}} \rho \underline{u} (\underline{u} \cdot \underline{n}) dA \cdot \underline{e}_x$$

$$= P_2 A_2 - P_1 A_1 + \int_{\partial A_{nozzle}} \rho U_x^2 \, dA$$

$\longrightarrow$

1) continued

Remember that in general, $Q = u A$, or

$$u = \frac{Q}{A}.$$

So:

$$\left(F_{nozzle}\right)_x = P_2 A_2 - P_1 A_1 + \int_{\partial A_{nozzle}} \rho \frac{Q^2}{A^2} \, dA$$

$$= P_2 A_2 - P_1 A_1 + \rho Q^2 \int_{\partial A_{nozzle}} \frac{1}{A^2} \, \partial A$$

$$= P_2 A_2 - P_1 A_1 + \rho Q^2 \left[\frac{1}{A_2} - \frac{1}{A_1} \right].$$

• watch the (-) signs!

Now you just plug in and solve.

$P_1 = 30 \, psig = 206,842 \, Pa$ $\qquad 1 \, Pa = 1 \frac{N}{m^2}$

$P_2 = 0 \, psig = 0$

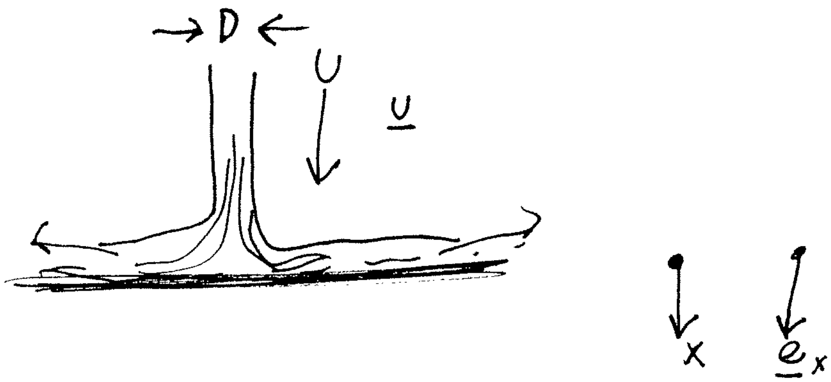
$A_1 = \pi (2 \, in)^2 = 0.0081 \, m^2$ $\qquad 1 \, m^2 = 1550 \, in^2$

$A_2 = \pi (1 \, in)^2 = 0.0020 \, m^2$

$Q = 40 \frac{gal}{sec} = 0.1514 \frac{m^3}{sec}$

$$\boxed{\begin{aligned} \left(F_{nozzle}\right)_x &= 6956 \, N \\ &= 1572 \, lb_f \end{aligned}}$$

2.) A Jet of fluid impinging on a plate



(a.) $$\sum F_x = \int_{\partial P} \rho\, \underline{u}(\underline{u}\cdot\underline{n})\, dA$$

$$= \int_{A_{in}} \rho\, \underline{u}(\underline{u}\cdot\underline{n})\, dA + \int_{A_{out}} \rho\, \underline{u}\,\cancel{(\underline{u}\cdot\underline{n})}^{\,0}\, dA$$

• in the cross section of A_{in}, $(\underline{u}\cdot\underline{n}) = (\underline{u}\cdot\underline{e}_x)$ = magnitude of $\underline{u}$.

• in the ring of fluid escaping away from the plate, $\underline{u}$ is $\perp$ to $\underline{n}$, so $\underline{u}\cdot\underline{n} = 0$.

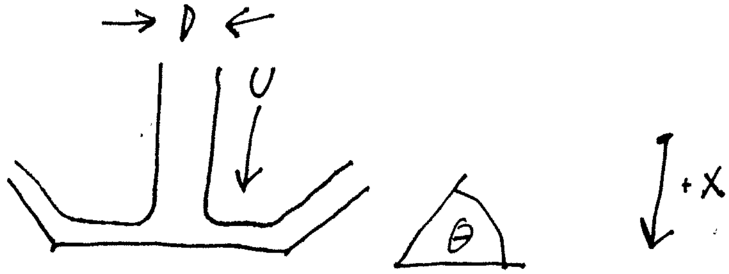
So:

$$F_x = -\rho U^2 A_{in}$$

$$\boxed{F_x = -\frac{D^2}{4}\rho U^2}$$

2.) continued

(b)



$$\sum F_x = \int_{A_{in}} \rho U_{x,in} (U)\, dA + \int_{A_{out}} \rho U_{x,out} (U)\, dA$$

- where $U_{x,in}$ = component of inlet velocity in x-direc
- $U_{x,out}$ = component of outlet velocity in x-direct

$$F_x = \rho U^2 A_{in} - \rho U A_{in} (-U \sin\theta)$$

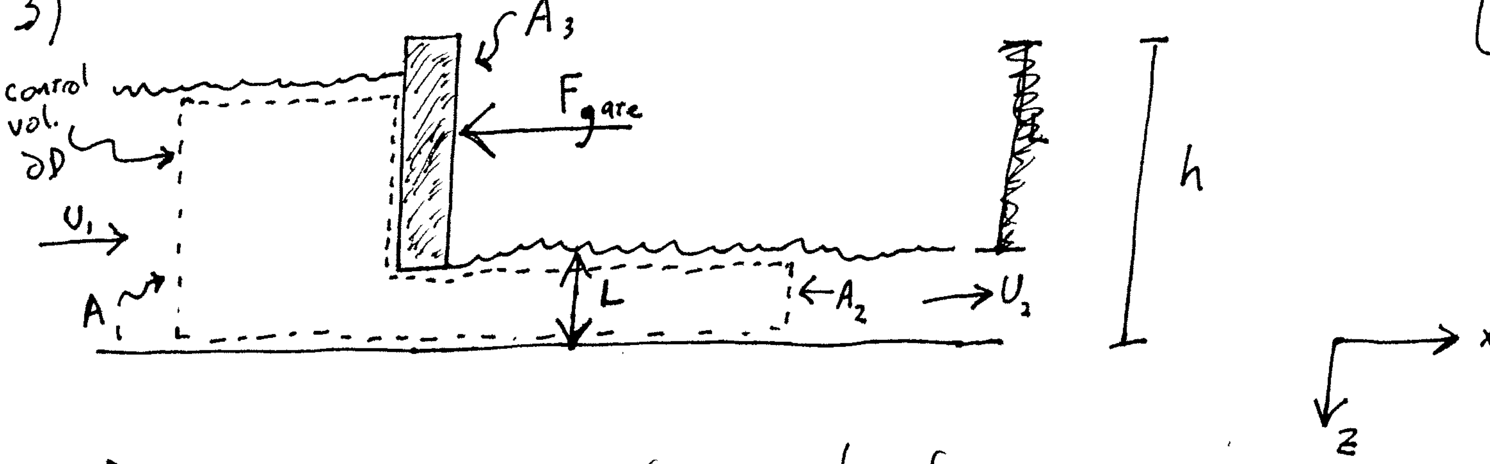
→ remember that from continuity: (cons. of mass)

$$Q = U A_{in} = U_{out} A_{out}$$

$$\boxed{F_x = \rho U^2 A_{in} (1 + \sin\theta)}$$

- this makes sense because the force must be greater for a curved plate than a flat plate.

3)



• Derive an expression for the force per unit width to keep the gate in place.

$$\sum \underline{F}_x = \int_{\partial D} \rho \underline{u} \, (\underline{u} \cdot \underline{n}) \, dA \cdot \hat{\underline{e}}_x \qquad \text{(Force balance)}$$

• First look at left-handside of the force balance.

$$\Sigma F_x = (F_1)_x + (F_2)_x + (F_{gate})_x$$

$$= \int_{A_1} -P \underline{n} \, dA \cdot \hat{\underline{e}}_x + \int_{A_2} -P \underline{n} \, dA \cdot \hat{\underline{e}}_x + (F_{gate})_x$$

– Let W = width of gate

– assume upstream & downstream pressure is hydrostatic.

$$= W \int_0^h \rho g z \, dz + W \int_{\cancel{(h-L)}\;0}^{L} \rho g z \, dz + (F_{gate})_x$$

$\longrightarrow$

3.) continued

$$\sum F_x = \frac{1}{2}\rho g h^2 W - \frac{1}{2}\rho g L^2 W + (F_{gate})_x$$

These terms are the hydrostatic pressure contributions to $(F_{gate})_x$.

- Now look at the right hand side of the force balance.

$$\sum F_x = \int_{\partial D} \rho \underline{u}(\underline{u}\cdot\underline{n})\, dA \cdot \hat{\underline{e}}_x$$

$$= \int_{A_1} (\rho u_1)\,(\underbrace{u_1 \hat{\underline{e}}_x \cdot (-\hat{\underline{e}}_x)}_{=-1})\, dA + \int_{A_2} (\rho u_2)\,(\underbrace{u_2 \hat{\underline{e}}_x \cdot (\hat{\underline{e}}_x)}_{=1})\, dA$$

$$= \int_{A_1} -\rho U_1^2\, dA + \int_{A_2} \rho U_2^2\, dA$$

$$= -\rho U_1^2 w h + \rho U_2^2 w L$$

Mass balance (steady state, incompressible)

$$Q = U_1 A_1 = U_2 A_2$$

$$= U_1 h \cancel{W} = U_2 L \cancel{W} \qquad U_2 = U_1 \left(\frac{h}{L}\right)$$

$\longrightarrow$

3.) Continued

$$\Sigma \underline{F}_x = -\rho U_1^2 wh + \rho U_1^2 \frac{h^2}{L^2} wL$$

$$= -\rho U_1^2 wh + \rho U_1^2 w \frac{h^2}{L}$$

Now go back to the force balance and substitute in both sides:

$$\Sigma \underline{F}_x = \int (\rho \underline{u})(\underline{u} \cdot \underline{n})\, dA \cdot \hat{\underline{e}}_x$$

$$\frac{1}{2} \rho g h^2 W - \frac{1}{2} \rho g L^2 W + (F_{gate})_x = -\rho U_1^2 wh + \rho U_1^2 w \frac{h^2}{L}$$

Rearrange

$$(F_{gate})_x = -\rho U_1^2 wh + \rho U_1^2 w \frac{h^2}{L} - \frac{1}{2} \rho g h^2 w + \frac{1}{2} \rho g L^2 W$$

$$= \rho W \left(-U_1^2 h + U_1^2 \frac{h^2}{L} - \frac{1}{2} g h^2 + \frac{1}{2} g L^2 \right)$$

$$\boxed{\frac{(F_{gate})_x}{W} = \rho \left[-U_1^2 h + U_1^2 \frac{h^2}{L} - \frac{1}{2} g h^2 + \frac{1}{2} g L^2 \right]}$$

This is the force on the gate per unit width.

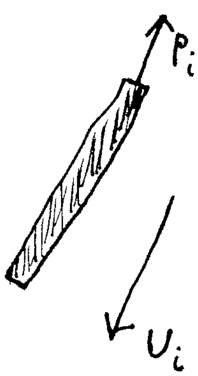
4.) Index notation

Cylinder settling in a viscous fluid

$$U_i = \left(\lambda_1 \delta_{iJ} + \lambda_2 p_i p_J\right) F_J$$

- λ_1 and λ_2 are constants independent of orientation.
- 2 mobilities given by:

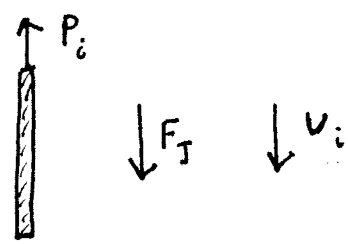
Parallel: $\dfrac{|U|}{|F|} = A$ Perpendicular: $\dfrac{|U|}{|F|} = B$



- A & B are determined experimentally.

- Find λ_1 & λ_2.

First look at parallel case.



$$\frac{|U|}{|F|} = A \Rightarrow U_i = A_{iJ} F_J = A F \delta_{i3} \qquad (1)$$

$$\left.\begin{aligned} F_J &= F \delta_{J3} \\ p_i &= \delta_{i3} \\ p_J &= \delta_{J3} \end{aligned}\right\}$$

I can do this because these vectors are all aligned in the 3-direction.

$\longrightarrow$

4.) continued.

$$U_i = A_{iJ} F_J$$

$$U_i = (\lambda_1 \delta_{iJ} + \lambda_2 P_i P_J) F_J$$

• but $F_J = F\delta_{J3}$, same with p's

$$= \lambda_1 \underbrace{F \delta_{iJ} \delta_{J3}}_{\delta_{i3}} + \lambda_2 F \delta_{i3} \underbrace{\delta_{J3}\, \delta_{J3}}_{\delta_{33} = 1}$$

so:

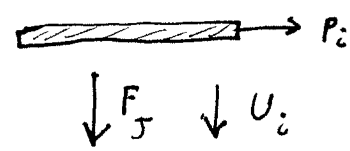
$$U_i = F(\lambda_1 + \lambda_2)\, \delta_{i3}$$

[look back at eq.(1)]

$$\Rightarrow \underline{A = \lambda_1 + \lambda_2}$$

Now perpendicular case:

$$\frac{|U|}{|F|} = B$$



$$U_i = B_{iJ} F_J = B F \delta_{i3} \qquad (2)$$

$F_J = F \delta_{J3}$ → same as before

$$\left.\begin{aligned} P_i &= \delta_{i1} \\ P_J &= \delta_{J1} \end{aligned}\right\}$$ aligned $\perp$ to the force.

→

4.) Continued

$$U_i = \cancel{B\,F\,\delta_{i3}} = B_{iJ}\,F\,\delta_{J3} \qquad [\text{from eq. (2)}]$$

$$U_i = \left(\lambda_1\,\delta_{iJ} + \lambda_2\,p_i\,p_J\right) F\,\delta_{J3}$$

$$= \left(\lambda_1\,\delta_{iJ} + \lambda_2\,\delta_{i1}\,\delta_{J1}\right) F\,\delta_{J3}$$

$$= \lambda_1\,\delta_{iJ}\,F\,\delta_{J3} + \lambda_2\,\underbrace{\delta_{J1}}_{\delta_{31}=0}\,\delta_{i1}\,F\,\delta_{J3}$$

$$= \lambda_1\,F\,\delta_{i3}$$

so $\boxed{B = \lambda_1}$

and $\lambda_1 + \lambda_2 = A$

$$\boxed{\lambda_2 = A - B}$$

$$\underline{U_i = \left(B\,\delta_{iJ} + (A-B)\,p_i\,p_J\right) F_J}$$