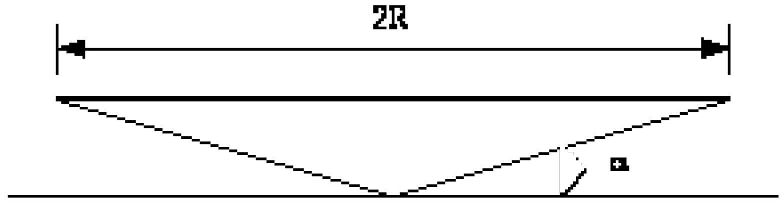
CHEG 355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Important: Most of the points for the first two questions will be awarded for setting the problems up correctly. Finish the problems off only after you've done the others!

Problem 1). (30 points) Lubrication: A cone of radius R and angle α is initially in contact with a plane as depicted below.



We want to use lubrication theory to determine the force F necessary to pull the cone off the plane with some velocity V in the limit of small α. In this limit, the gap width is simply αr. (Hint: if R is the length scale in the radial direction, what is the length scale in the z-direction???)

a. The radial momentum equation and continuity equation are given below. Making use of symmetry, eliminate terms which are identically zero, and write down the equations and boundary conditions which govern this problem.

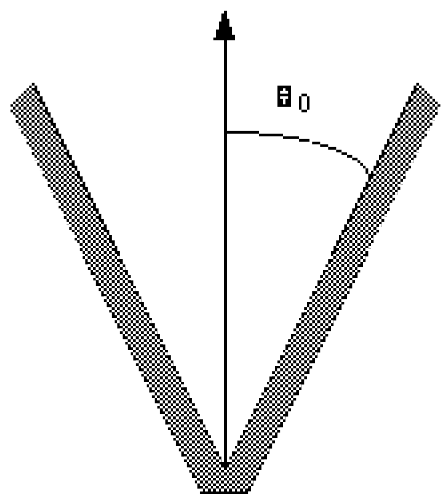
$$\frac{1}{r}\frac{\partial\left(r\,v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

b. Render the equations dimensionless in the lubrication limit, and show which terms are important. By determining the scaling for pressure (and rendering the equation for the required force dimensionless as well), show how the force scales with the parameters R, V, μ, and α.

c. Solve the problem.

Problem 2). (30 points) Two Dimensional Flows: Consider the two-dimensional geometry depicted below. A very viscous fluid is contained in a triangular trough (angle $2\theta_0$), where the walls are not moving. An elastic sheet is centered in the middle of the trough ($\theta = 0$). We want to calculate the streamfunction for the fluid flow resulting from the stretching of the elastic sheet. Note that this stretching yields the boundary condition $u_r \Big|_{\theta=0} = \lambda r$, e.g., the radial velocity of the sheet is proportional to r.



a. Set up the problem as completely as possible, with all boundary conditions clearly determined.

b. Using the boundary conditions as a guide, determine the form of the streamfunction, and develop the corresponding ODE and boundary conditions.

c. Determine the general solution to the ODE, and get a set of equations for the unknown coefficients.

d. Explicitly obtain the coefficients for the specific case where $\theta_0 = \pi/2$.

Hint: The solution to this problem is quite similar to the collapsing wedge problem you solved for homework last week!

Remember that in the cylindrical geometry we have:

$$v_\theta = \frac{\partial \psi}{\partial r} \quad , \quad v_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Problem 3. (30 points) Short Answer

a. (20 pts) Briefly identify the physical mechanism described by each of the following terms:

1. $\mu \dfrac{\partial^2 u_x}{\partial y^2}$

2. $\dfrac{D\rho}{Dt}$

3. $-\rho \dfrac{v_\theta^2}{r}$

4. $\rho \dfrac{v_r v_\theta}{r}$

5. $E^4 \psi = 0$

6 - 10. (each term): $\rho \dfrac{\partial u_i}{\partial t} + \rho u_j \dfrac{\partial u_i}{\partial x_j} = -\dfrac{\partial p}{\partial x_i} + \mu \dfrac{\partial^2 u_i}{\partial x_j^2} + \rho g_i$

b. (10 pts) Multiple Choice:

1. In ship modelling we try to preserve which dimensionless number:
   A. Reynolds Number
   B. Prandtl Number
   C. Froude Number
   D. Weissenberg Number

2). Which (if any) of the following can be discontinuous at a fluid-fluid interface?
   A. Shear stress
   B. Heat flux
   C. Mass flux
   D. Velocity

3). The "Big Bang" is probably the ultimate transport problem. Current theories ascribe the homogeneity of the universe to a period of "inflation" following the initial "Bang". The period of inflation lasted approximately
   A. 10^{-32} seconds
   B. 10^{-3} seconds
   C. 10 seconds
   D. 10^{9} years

4). Which of the following operate in an inertia dominated regime? (give all that apply)
   A. A car
   B. A bicycle
   C. A minnow
   D. A LabChip

5). Estimate the Reynolds number of a swimmer who swims a kilometer in 20 minutes.