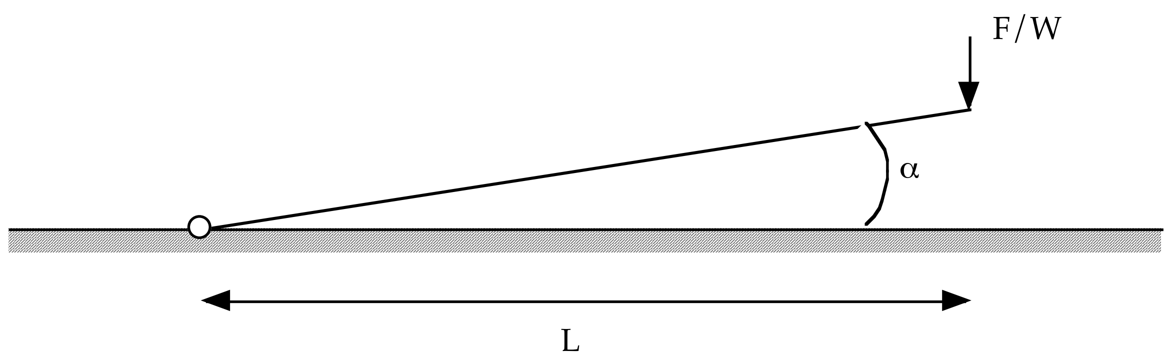
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Lubrication: A hinged plate of extension W into the paper and of length L as depicted below is being squeezed shut with some force F/W. We wish to determine the closure rate $\Omega = - d\alpha/dt$ as a function of the parameters of the problem for small angles $\alpha << 1$.

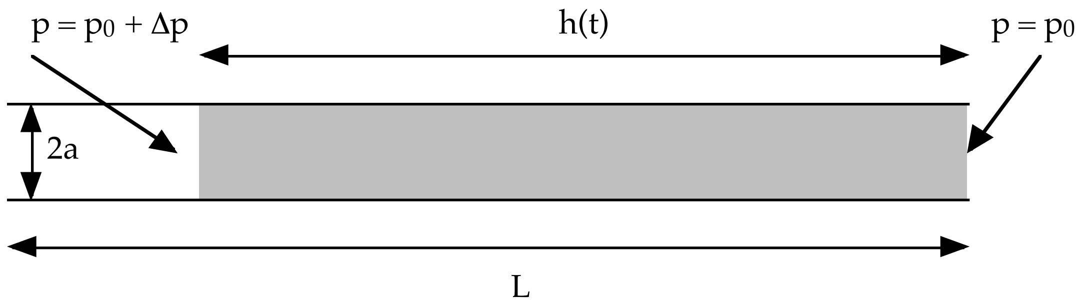


a. Write down the governing differential equations, boundary equations, mass balance, torque balance about the hinge, etc., necessary to solve the problem in the lubrication limit.

b. Using scaling analysis of the equations in part (a), determine how Ω depends on the parameters of the problem to within an unknown constant.

c. Solve the problem. You may leave the final result for the torque balance in an integral form if you wish.

Problem 2. (30 pts) Scaling/Unidirectional flows: Consider a horizontal straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below. The liquid is initially at rest. At time t=0 we start to blow the liquid out of the straw by applying a constant pressure differential Δp. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw in both the high and low Re limits - e.g., how long does it take for h to reach zero.



a. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

c. Solve the problem to obtain the dimensionless non-linear second order ODE which governs the evolution of the liquid slug length in this limit, together with initial conditions. This equation is trivial to solve numerically using matlab, of course, (the dimensionless blowout time comes out to be (π/2)^.5) but don't do it here!

d. Scale the equations for LOW Reynolds numbers, and determine the new (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

e. Solve for the actual drainage time in the low Reynolds number limit (e.g., solve the dimensionless equations obtained in part d to get the numerical value).

The following equations may be helpful:

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

Problem 3). (20 points) Short Answer / Multiple Choice:

(2 pts each) Briefly identify the physical mechanism (or equation name) described by each of the following terms **and a problem where it would play a role**:

1. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

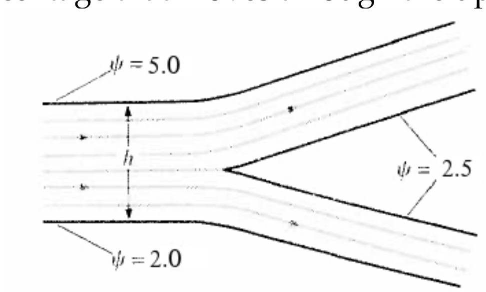
2. $\nabla^4 \psi = 0$

3. $\rho \dfrac{v_r v_\theta}{r}$

4. $p + \frac{1}{2} \rho u^2 + \rho g h = Cst$

5. It is proposed to model the flow patterns in a microfluidic chip of characteristic channel depth of 100μm and velocity of 1cm/s with a macroscopic scale model of channel depth 2mm. If the working fluid of the chip is water, and that of the model is a mixture of glycerin and water with kinematic viscosity 0.1cm^2/s, what should the velocity of the fluid in the model be?

6. Computer fluid simulations were carried out in a 2-D asymmetric channel bifurcation as presented below. The simulation reveals the stream function values at the channel walls. What is the flow percentage that moves through the upper branch of the channel?



7. The symmetric part of the rate of strain tensor is usually given the symbol e_{ij}. What is its representation in terms of u_i? (Index notation, please!)

8. Prove that inertial effects always increase the drag on an object relative to that caused by viscous effects alone. Be brief!

9. A small jellyfish is about 5cm across and moves at a velocity of about 1cm/s. Estimate its Reynolds number. Is its motion dominated by inertial or viscous effects?

10. In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 10m boat of this type?