CBE 30355 Transport Phenomena I
Final Exam

December 13, 2010

Closed Books and Notes

Problem 1. (20 points) Scaling/Boundary Layers: For homework you examined the boundary layer produced by a rotating disk. In this problem we use this velocity distribution to determine the temperature of a heated, rotating disk. Consider the disk of radius R depicted below, rotating with angular velocity Ω in an unbounded fluid. The velocity distribution in the r and z directions (the ones that matter) near the disk are given by:

$$u_r = \lambda\left(\frac{\Omega^3}{\nu}\right)^{1/2} z\,r \qquad ; \qquad u_z = -\lambda\left(\frac{\Omega^3}{\nu}\right)^{1/2} z^2$$

where λ is an O(1) dimensionless constant that would be obtained from a numerical solution of the momentum transfer boundary layer equations. The disk is heated, such that at the surface there is a constant heat flux (energy/area/time) of:

$$q_z\big|_{z=0} = -k\frac{\partial T}{\partial z}\bigg|_{z=0} = q_0 \quad (const)$$

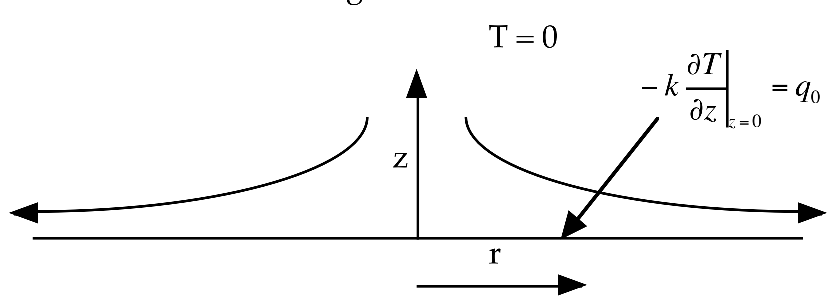
a). By scaling the energy equation and boundary conditions in the boundary layer limit, determine the characteristic temperature of the surface of the disk as a function of the parameters of the problem. You may take the temperature far away from the disk to have a reference value of zero (just like pressure in incompressible flow, the absolute value of the temperature doesn't matter as long as material properties are assumed constant). The energy equation for this problem is:

$$u_r\frac{\partial T}{\partial r} + u_z\frac{\partial T}{\partial z} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right]$$

b). How fast do we have to rotate the disk for the scaling result obtained in part (a) to be valid?

c). Using affine stretching, show that this (now dimensionless) problem admits a self-similar solution. Obtain the corresponding ODE and boundary conditions.

d). Solve the equation to determine the temperature distribution on the disk. You may leave the final result in terms of an integral.



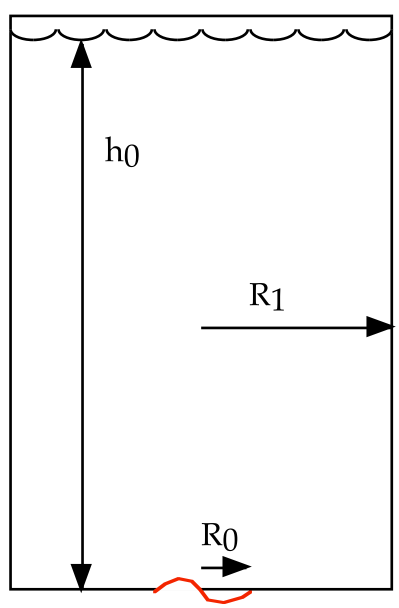
Problem 2. (20 points) Earlier this semester students demonstrated the interesting issue of the drainage of a can of liquid through a small hole of radius R_0. Here we examine this phenomenon:

a). If the top of the can is sealed (only the one hole in the bottom!) it is observed that the liquid won't come out at all if the radius of the hole is sufficiently small, and that the critical size doesn't depend on the height or radius of the can either. Adding a tiny amount of soap to the hole caused the liquid to start running out, showing that the drainage was held off by surface tension. Using these observations and dimensional analysis, **estimate** the critical radius R_0 for water ($\Gamma = 70$ dyne/cm).

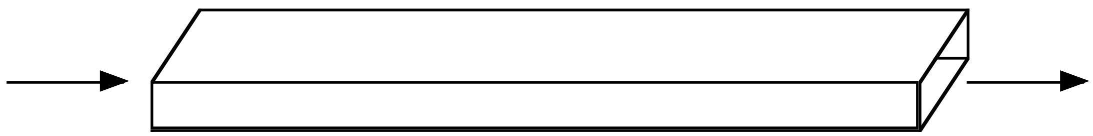
b). Physically, why shouldn't the critical size of the hole depend on the height of the fluid in the can? No more than a couple of sentences!

c). When a hole was punched into the top of the can, air rushed in and the liquid drained out very quickly. Neglecting viscosity, develop an equation to determine the velocity and flow rate as a function of fluid depth, and use it to calculate the drainage time for initial height h_0. Evaluate this drainage time for the properties of water, $h_0 =$ 10cm, $R_1 = 5$cm, and $R_0 = 0.25$cm. You may want to use appropriate "K" values from the table on the next page!

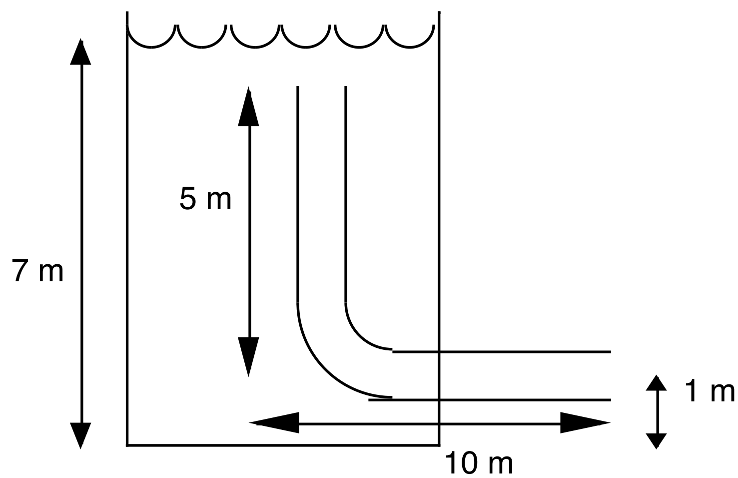
d). A colleague argues that you are making a serious mistake by ignoring viscous effects. Estimate how large the viscosity of the fluid in the can would have to be to significantly affect the drainage time.



Problem 3. (10 points) Plane Poiseuille Flow: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is 50μm deep, 1mm wide, and 2cm long. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 1mm x 2cm surface - where cell adhesion is being studied) of 10 dyne/cm^2. If the working fluid has the same viscosity as water, what should the flow rate of the pump supplying the fluid be?



Problem 4. (10 points) You are designing an overflow drain for a tank as depicted below. It is required that the pipe must handle a flow rate of 5 liters/s. What is the minimum diameter of the drain pipe?



You may find the following expressions useful:

$$h_L = \frac{\langle u \rangle^2}{2g} \Sigma K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \mathrm{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

Problem 5. (30 points) Pump Curves / Additional Readings / Short Answer:

The first seven questions refer to the pump curve below:

1. It is desired to pump 75 liters/sec from a pond to an elevation of 75 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH150 recommended for the job?

2. What is the useful mechanical work done <u>by</u> the pump <u>on</u> the fluid per unit time?

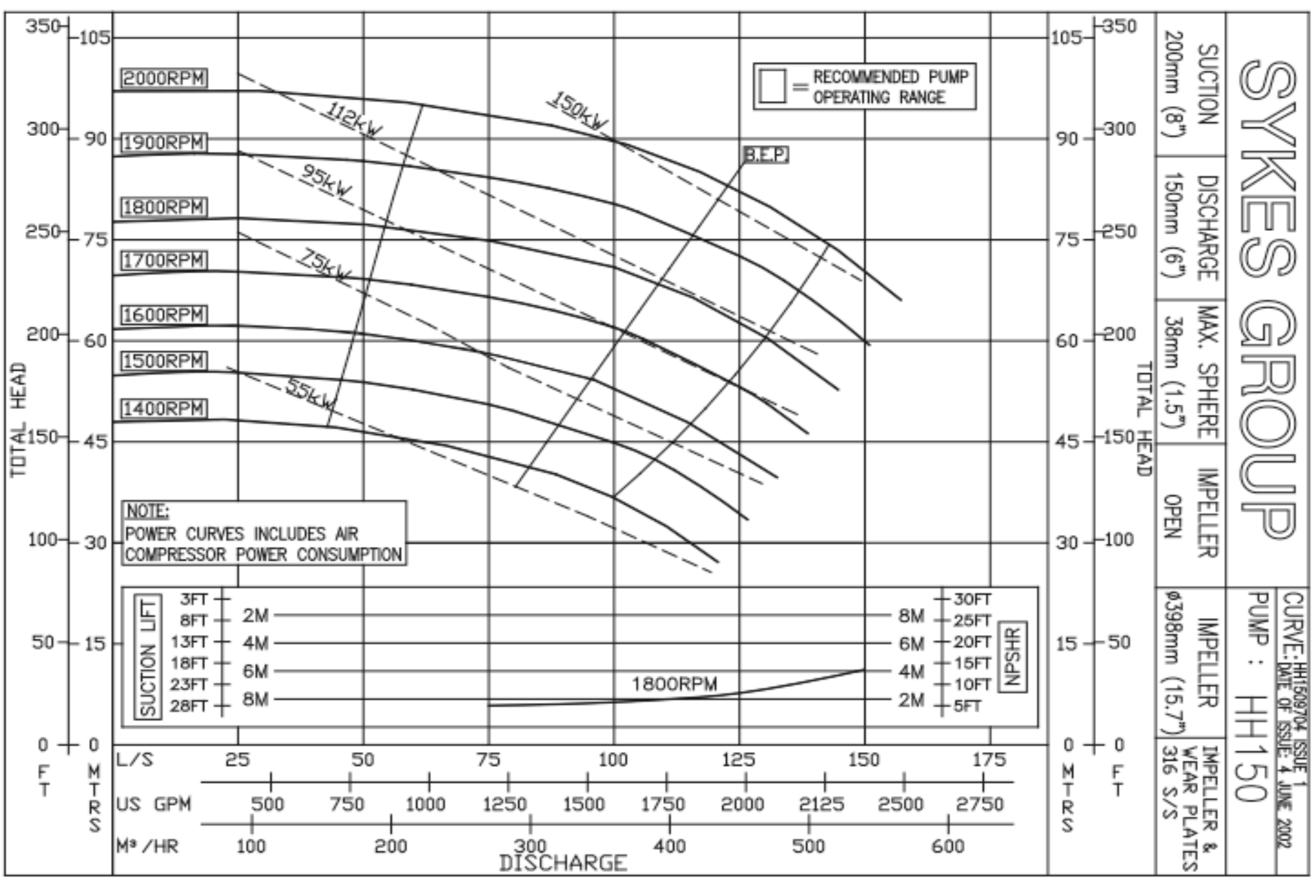
3. What is the efficiency of the pump at the operating conditions?

4. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

5. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

6. Your boss proposes to use a 10cm diameter pipe for this system. Quantitatively demonstrate why this is probably a bad idea.

7. About how big should the pipe be instead? Make any approximations you think are reasonable.



8. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

9. A sphere of density ρ_m (> fluid density ρ_f) and radius a is sitting on the wall of a microfluidic channel of half-width b (b>>a). At high enough velocities U, inertial lift effects will overcome gravity and cause the sphere to lift off from the wall. **Estimate** the critical velocity. (Hint: in the limit b>>a the lift scales with the shear rate at the wall, not directly with the mean velocity.)

10. Why can't viscous forces cause the sphere to lift off from the plane in channel flow? No more than two sentences!

11. At high Re, drag principally results from:

A. Potential Flow
B. Boundary Layer Separation
C. Skin Friction
D. Turbulence

12. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

13. Experimentally, how can you most accurately calculate the shear stress on a plate in boundary layer flow at high Reynolds numbers from the velocity profile?

14. For a shear stress of 49 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

15. Using index notation, write down the force exerted by the surrounding fluid on an arbitrary object in terms of the stress tensor σ_{ij}.