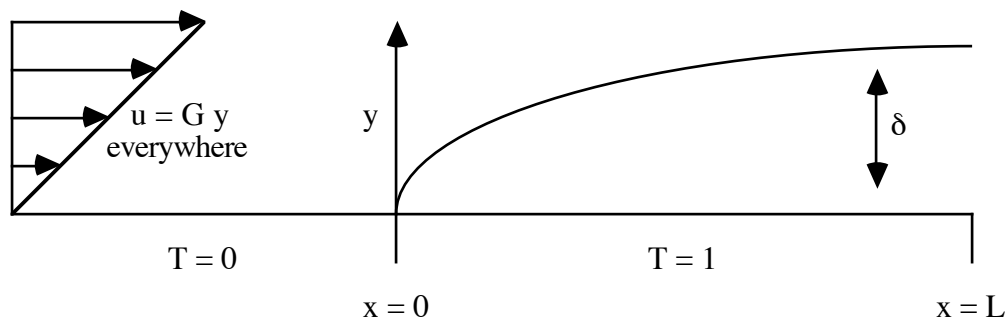1. Last week you scaled the problem of shear flow past a heated plate.   This time you will solve a somewhat simpler version of it!

a). By using the coordinate stretching technique illustrated in class, show that the boundary layer problem described below admits a similarity solution and obtain the similarity rule and similarity variable. Obtain the transformed ODE and boundary conditions. How does the thickness of the thermal boundary layer grow as it moves down the plate?

b). Solve the ODE. Note that $f''/f' = (\ln(f'))'$. You may leave the final result in terms of an explicit integral of a known function, or you may evaluate the integral in terms of the incomplete gamma function (you can look it up in a handbook, or online). Obtain a similar explicit relationship for the heat loss from the plate as a function of the length of the plate. Note that nearly all aspects of the solution except the final numerical value may be learned without explicitly solving the equation.

Recall that the problem was flow past the flat plate y=0 as depicted below. Fluid with a dimensionless temperature $T = 0$ flows along the plane in the x direction with velocity $u = Gy$ where G is the shear rate (e.g., plane Couette flow without the upper plane). The plate is maintained at a temperature $T = 0$ for $x < 0$ and a temperature $T = 1$ for $x > 0$. The governing equations and boundary equations are given below.

$$T = 0$$

$$G y \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$T \bigg|_{y=0,\, x<0} = 0 \qquad T \bigg|_{y>>0} = 0 \qquad T \bigg|_{y=0,\, x>0} = 1 \qquad Q = \int_0^L -k \frac{\partial T}{\partial y} \, dx$$

2. Boundary layer growth with suction: One technique used to control the rate of boundary layer growth on airplane wings is suction -- the wing (or plate) is porous, and fluid is sucked out of tiny holes which has the effect of keeping the boundary layer attached and preventing separation. In this problem we will examine the simple case of

uniform flow past a flat plate where the vertical suction velocity is given by the power-law relation:

$$v\big|_{y=0} = -\lambda\, x^{-1/2}$$

a. What should be the characteristic magnitude of λ to affect the boundary layer thickness (e.g., how should it scale with U, μ, ρ, L, etc.) and what should be the magnitude of the total amount of gas withdrawal (the integral of v over the plate)?

b. Solve for the dimensionless displacement thickness and wall shear stress ($f''(0)$) as a function of λ^* (e.g., λ divided by its characteristic scaling) and plot it up. Note that this will require a numerical solution to the Blasius Equation - where your boundary condition $f(0) = 0$ is replaced by one which involves λ^*.
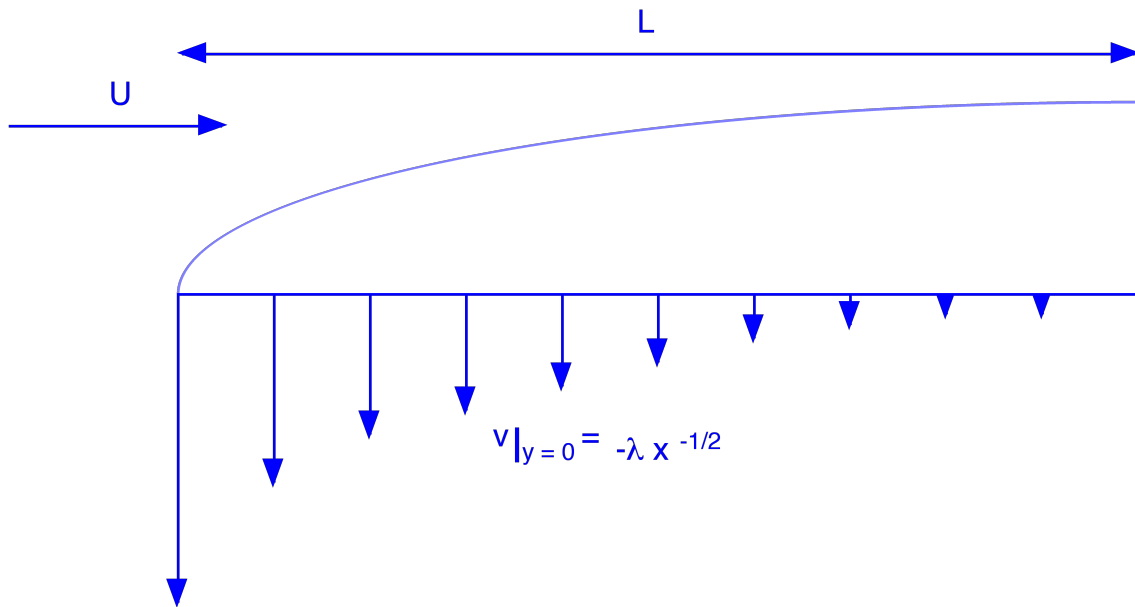
3. An important experimental geometry in electrochemistry is the spinning disk electrode, in which a disk is spun rapidly in a fluid. The diffusion of momentum away from the surface of a disk imparts a centrifugal force which throws the fluid out radially, drawing fluid in axially. All three components of the velocity are non-zero (the coriolis force matters too), however some components are larger than others. Here we examine this problem in the boundary layer limit.

Consider a disk of radius R spinning with angular velocity Ω in an infinite fluid at rest. By scaling the r, θ and z momentum equations, estimate the boundary layer thickness and characteristic radial velocity as a function of the parameters in the problem. What dimensionless group must be small in order for the boundary layer scalings to apply?