PROBLEM SET 2 CBE30355 DUE 9/9/14

1). Compute the viscosity of air from 100ºK to 500ºK and 1 atm pressure. The Chapman-Enskog equation described in chapter 1 of BS&L and the data in tables E.1 and E.2 are useful here (note the correlation at the bottom of table E.2)! Compare your result graphically with the data from:

http://www.engineeringtoolbox.com/air-absolute-kinematic-viscosity-d_601.html

2). The natural log of the viscosity of many liquids is approximately quadratic in the inverse of the temperature in ºK (e.g., the equation in chapter 1 of BS&L, with an extra term).

a. Using this, and data from the web page:

http://www.engineeringtoolbox.com/dynamic-viscosity-motor-oils-d_1759.html

determine constants for such a model for the different grades of motor oil (plot the correlations and the data up using a semilog scale).

b. What are the temperature coefficients for the different grades at 25°C? This is the fractional change in viscosity per degree centigrade.

c. It is suggested that intermediate grades of motor oil can be regarded as a mixture of SAE10 and SAE50 oils. The viscosity of a mixture of two miscible liquids is roughly the volume fraction weighted geometric mean of their individual viscosities (this is a lot more shaky than the temperature relationship, but works pretty well for simple fluids that don't have interesting chemical interactions going on). Using this idea, determine the blend ratio of SAE10 and SAE50 necessary to get an oil with the properties of SAE30, and graphically compare the expected viscosity of the blend to the data for SAE30.

By the way, multigrade motor oils are made not by blending two oils, but rather by taking a low viscosity grade and adding a polymer to it. The polymer has the property of reducing the temperature coefficient, improving performance at high temperatures while only slightly increasing the viscosity at low temperatures.

3). Here's a weird application of hydrostatics: Consider a ball of gas (air) floating in space far from any other source of gravity (tidal orbital dynamics would really mess this up!). At the center of the ball, we take the pressure to be 1 atm and the temperature to be a balmy 20°C. As we move outward from the center, the pressure decreases by hydrostatics and the temperature drops by adiabatic expansion (e.g., it obeys both the ideal gas law and PV^{γ} adiabatic expansion). This is the result for a "well mixed" atmosphere, and applies to the earth's atmosphere (at least below the stratosphere, anyway) as well. Our goal is to determine the mass of the ball of gas.

a. Set up the problem as a pair of equations for total mass inside a particular radius and density as a function of position using hydrostatics, the adiabatic gas law, and mass conservation. Don't forget that gravity is a function of position, and that the gravitational attraction *inside* a spherical shell is identically zero!

b. Scale the mass and radius by some unknown values, and then use the equations to determine the characteristic mass and radius (e.g., the magnitude of the scaling parameters so that the resulting dimensionless equations will be of O(1).

c. Solve the dimensionless problem numerically to determine the final value of the mass of gas, and compare it to the mass of the earth. It's pretty easy to set the dimensionless problem up as a pair of coupled first order non-linear ODE's.

4). Pool drains can be dangerous things - there was a tragic case a few years ago in this area where a child was stuck in a drain on the bottom, plugging it, and drowning as a result. Here we look at a somewhat simpler problem. Suppose a ball of radius R is plugging a drain of diameter D at the bottom of a pool of depth h as depicted above. Obviously, $R > D/2$ or the ball goes down the drain! *Estimate* the conditions under which the net force on the ball is zero for very small ratios of D/R (you can do the *precise* calculation for arbitrary D/R, but the math gets a little messy!). Assume that the pressure distribution in the drain is just atmospheric pressure, and that in the water is governed by the hydrostatic pressure distribution. If R is 1 ft and D is 4 inches, what is the corresponding depth?