1). Consider the geometry below. Two sheets are drawn with a velocity U between two rollers of radius R whose surfaces are separated by a distance 2 b_0. The idea is to fill the space between the sheets with a viscous fluid of viscosity μ, and all the action takes place in a lubrication layer between the rollers, at least in the limit $b_0/R \ll 1$. Here we analyze this problem.

a. By scaling the flow equations in the gap (use Cartesian coordinates!), show how the force/width on the rollers F/L scales with the separation distance between the rollers and the other parameters of the problem.

b. The sheet sandwich detaches from the rollers downstream at a point x_d and final gap width 2 b_f > 2 b_0 when the pressure again returns to zero. Develop an implicit integral relation for the ratio b_f/b_0, and show that it doesn't depend on any of the other parameters of the problem.

c. Develop an integral relationship for the dimensionless pressure and force and evaluate it numerically.

Hint: the gap geometry in the lubrication limit is given by $b = b_0 + \frac{1}{2}\frac{x^2}{R}$ where x is the distance along the gap from the point of minimum separation. By continuity, the flow rate per extension into the paper (Q/W) through any vertical plane -must- be the same, and must equal the exit flow rate $U*2b_f$.
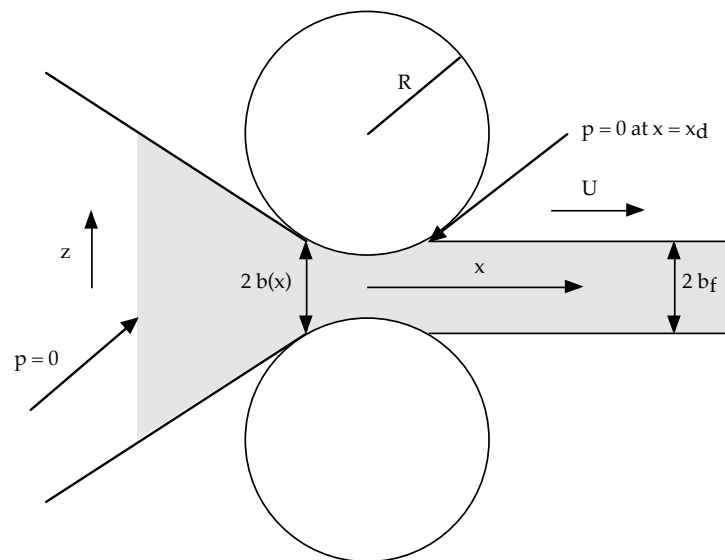
2. Here we examine the sliding block considered in class:

a. If the block dimensions are L = 10cm, W = 30cm, and d_2-d_1 = 0.1cm (e.g., a fixed inclination angle), determine the velocity U at which the block can support a weight of 10kg while maintaining a minimum separation d_1 = 0.01cm. Take the fluid viscosity to be 1.0 poise.

b. By examining the shear stress on the plate (you can get this from the velocity distribution in the gap), determine the force necessary to move the block with this velocity.

3. Thermodynamics and Scaling Analysis: In this problem we estimate the temperature and fluid velocity in a chimney as a function of various physical parameters. The chimney is of cross-sectional area A and height H, thus the total volume of air in the chimney is AH. We have a source of energy at the bottom given by Q (assumed to be distributed over the whole area). The energy, of course, raises the temperature of the air, causing it to expand. Using the principles of conservation of energy and momentum, and neglecting all frictional and heat losses, estimate the velocity and temperature of the air in the chimney. Hint: You are going to have to remember some of your thermodynamics from last term to get this one!

4. Dimensional Analysis of a mixer: You are trying to mix two fluids with slightly different densities in a large mixer. Empirically, you find that a vertical mixing velocity of 10 cm/s in a 1m high mixer suffices to mix water and a solution with density 1.01 g/cm^3. How large should the vertical mixing velocity be for a 2m high mixer to mix water and a solution with density 1.04 g/cm^3? What dimensionless group should you try to preserve?