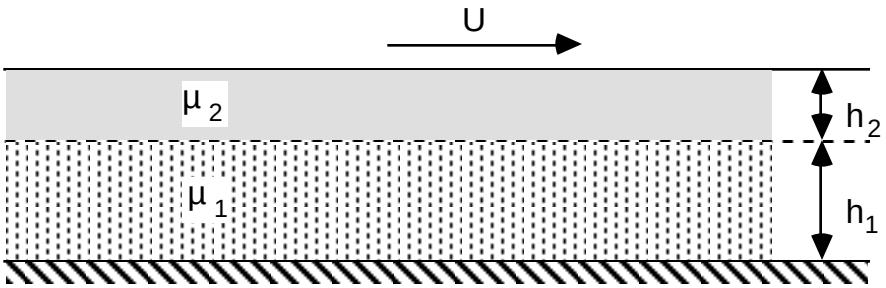
CHEG 355 TRANSPORT PHENOMENA I

First Hour Exam
10/9/01

This test is closed books and closed notes

Problem 1. (20 pts) Consider the stratified fluid depicted below which is being sheared between two infinite parallel plates. If the upper plate is constrained to move with velocity U, the lower plate is fixed, and the viscosities and thicknesses of the two layers are μ_1 and μ_2 and h_1 and h_2, respectively, calculate the following:

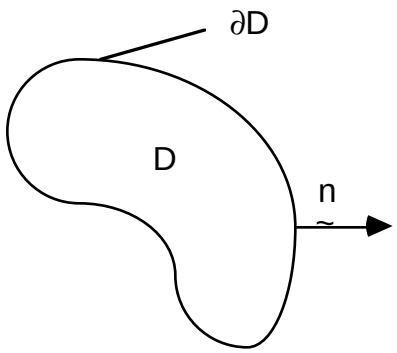


a. Write down the Navier Stokes equation for momentum *in the direction of flow* in *each* region and show which terms are non-zero.

b. What are the velocity and shear stress distributions? What is the force on the lower plate? Remember that both velocity and shear stress *must be continuous functions of position!* This determines the conditions at the interface between the two fluid layers.

c. Calculate the flow rate of each fluid (per width W out of the plane of the paper).

Problem 2. (20 pts) Consider the arbitrary fluid element depicted below. If we have a flow containing several species which are undergoing reaction (a source/sink per unit volume) and diffusion (a flux of each species *in addition to convection*), derive the equation which governs the conservation of each species. The source of species i due to reaction will be denoted R_i (units of mass of i per unit time per unit volume) and the *total* mass flux of species i (diffusion *and* convection) is given by $(\rho_i \mathbf{u} + \mathbf{j}_i)$, in which ρ_i is the mass of species i per unit volume, $\mathbf{u}$ is the total mass-average velocity of the fluid, and $\mathbf{j}_i$ is the diffusive flux of species i. Note that both $\mathbf{u}$ and $\mathbf{j}_i$ are *vectors*. We are *not* using index notation in this problem!

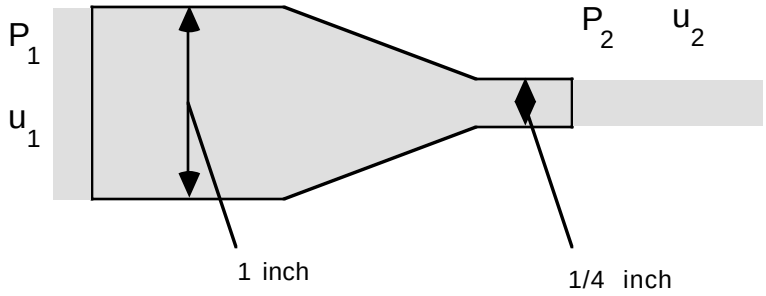


a. Write down in words the law governing the conservation the mass of species i for the control volume D. Don't forget diffusion and reaction.

b. Write down the integral relationship equivalent to that in part a.

c. Through application of the divergence theorem, obtain a microscopic equation valid at every point in the flow. This will still be in terms of $\mathbf{j}_i$ (we have not specified a constitutive relation here for diffusion, analogous to Newton's law of viscosity or Fourier's law of heat conduction - don't worry about it).

Problem 3. (20 pts) Consider the hose nozzle depicted below. You are asked to determine the force exerted by the fluid on the nozzle in order that the hose-nozzle coupling may be properly designed. The inlet to the nozzle is 1 inch ID, the outlet (and emerging jet) is 1/4 inch ID, and the water pressure in the hose before the nozzle is 40psig (2.75×10^5 N / m^2 above atmospheric pressure). Neglect any forces due to gravity.



a. Develop an equation for the force on the nozzle in terms of u_1, u_2, P_1, P_2 and the two areas. What key assumption are you making here?

b. Taking P_2 = 0 psig and employing conservation of mass, reduce the above result to one involving only P_1, u_1, and the areas.

c. If we neglect frictional losses of mechanical energy, the velocities and pressures are governed by Bernoulli's equation:

$$\frac{1}{2}\rho u_1^2 + P_1 = \frac{1}{2}\rho u_2^2 + P_2$$

Use this relation to get a quantitative answer for the force on the nozzle.

Problem 4. (10 pts) Index notation. Write the following expressions using index notation:

a. $A = \underset{\sim}{\nabla} \cdot \underset{\sim}{u}$ (dot product)

b. $\underset{\approx}{A} = \underset{\sim}{\nabla}\, \underset{\sim}{u}$ (vector composition product)

c. $\underset{\sim}{\omega} = \underset{\sim}{\nabla} \times \underset{\sim}{u}$ (curl)

d. $\underset{\sim}{C} = (\underset{\sim}{x} \cdot \underset{\sim}{y})\, \underset{\sim}{z}$

e. $\underset{\approx}{A}^T \underset{\approx}{A}\, \underset{\sim}{x} = \underset{\approx}{A}^T \underset{\sim}{b}$

Problem 5. (10 pts) Additional Readings questions - just list which answer is the correct one!

1. In what city and at what time of year did the Great Molasses Flood occur?
   A. Boston, Spring
   B. Philadelphia, Summer
   C. Boston, Winter
   D. New York, Fall

2. What fraction of an iceberg's mass is below the waterline when it is floating free in the ocean?
   A. 1/3
   B. 1/2
   C. 3/4
   D. 7/8

3. Small variations in initial conditions sometimes result in huge, dynamic transformations in concluding events. This statement summarizes the idea behind:
   A. The Peanut Butter Effect
   B. The Butterfly Effect
   C. Poiseuille flow
   D. The Prandtl Number

4. A dam up river of what West Virginia city failed on May 31, 1889?
   A. Charleston
   B. Smithville
   C. Rivendell
   D. Johnstown

5. Fluidized mixtures of solid and semi-solid fragments, and hot expanding gasses which flow down the sides of volcanoes are known as:
   A. Pyroclastic flows
   B. Igneous flows
   C. Sediment flows
   D. Mudslides