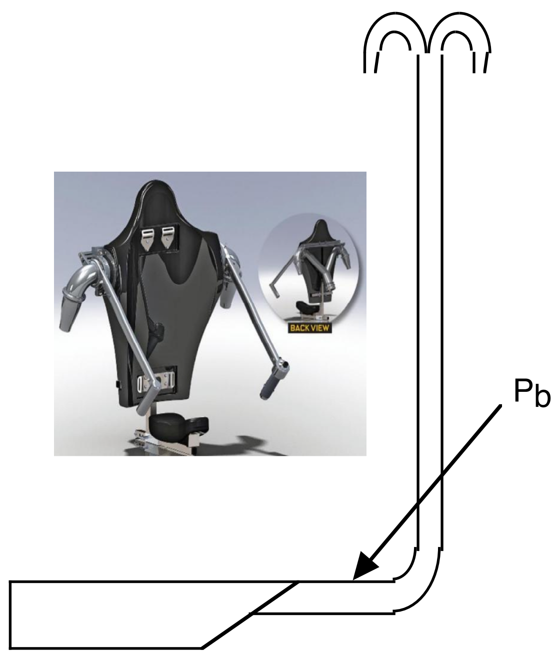
CBE 30355 TRANSPORT PHENOMENA I

First Hour Exam
10/6/15

This test is closed books and closed notes

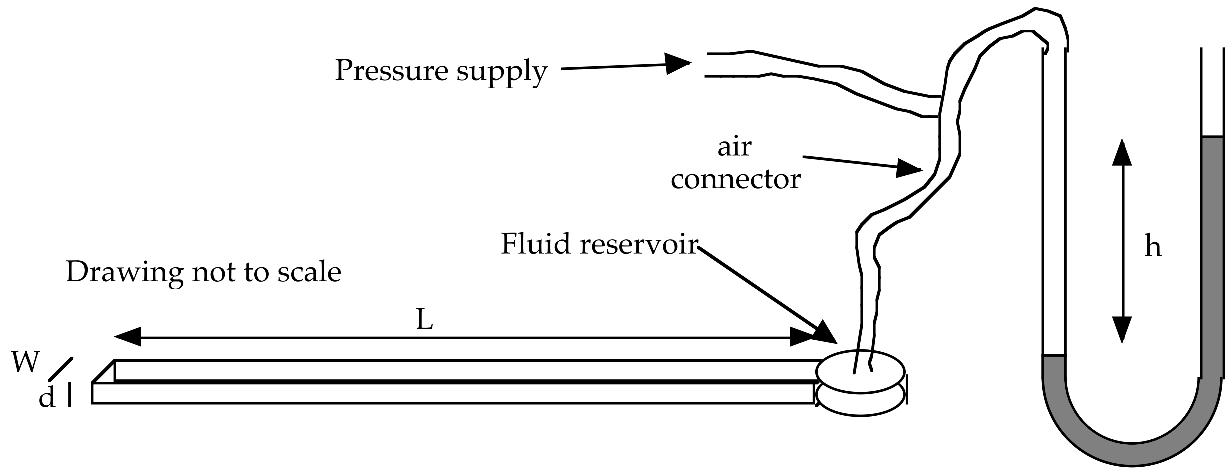
Problem 1. (20 pts) Conservation of Momentum: In a fun WSJ article on water toys for the super-rich, I came across the Jetlev Flyer. It consists of a small boat/pumping unit, a 10m long 4” ID feed tube, and a jet backpack which redirects the water downwards through two 1.5” ID nozzles. According to their advertisements, you can fly for hours (before the modified jetski boat pump runs out of gas), and travel up to 25mph towing the pump boat behind you. Although more than a bit pricey (it costs around the same as a new car!) you can actually get a 15 minute ride on one for less than $200 at a bunch of places. Here we analyze the requirements for the pumping system.



Flyer weight: 100 kg
Jetpack weight (water filled): 10 kg
Feed pipe dry weight: 0.25 kg/m

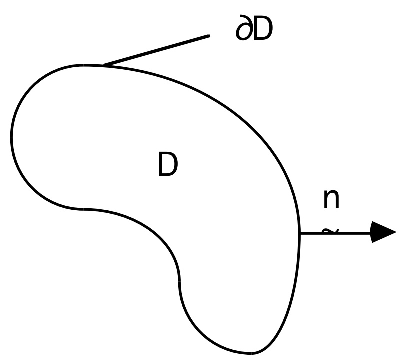
Using this information, and neglecting all frictional losses (e.g., assume that Bernoulli’s equation applies), determine the required flow rate, and the pressure at the **base** of the feed pipe (e.g., the pressure the pump would have to supply) to support our flyer at the maximum altitude of 10m. Don’t forget the weight of the water in the feed pipe! While the flow rate you calculate is about right, the required pressure (and pump power rating) would be significantly higher due to losses as we shall study later this term.

Problem 2. (20 pts) Unidirectional Flow: In our laboratory we have studied pressure-driven flow in microfluidic channels, and in some cases monitored the pressure using an open ended water manometer such as discussed in class. Consider the simplified system depicted below:



The channel dimensions are 50μm deep by 1mm wide by 5cm long, and the working fluid for both the manometer and the channel is water. If the manometer reading was 10cm H_2O (about as high as we could go before the manometer blew out), what would be the flow rate? Give the answer in microliters/minute. Hint: for unidirectional flow in any conduit, the flow rate is controlled by viscous diffusion of momentum in the thinnest direction.

Problem 3). (20 pts) Beginning with an integral momentum balance for the arbitrary control volume depicted below, derive the Cauchy Stress Equations (e.g., the momentum equations in terms of the total stress tensor σ_{ij}). Leave your final result in terms of the material derivative of the velocity. You may use index notation or Gibbs notation (ordinary vector notation) as you prefer.



Problem 4. (20 pts) Short Answer

1. Provide the mathematical representation of the Reynolds number in terms of physical parameters and explain its significance.

2. What is the ratio of the centerline velocity to the average velocity for a) laminar flow in a tube, and b) laminar flow in a channel?

3. What is the continuum hypothesis, and where does it break down?

4. Write down Bernoulli's Equation. What is it a statement of and what does it ignore?

5. Write down the continuity equation for a c**ompressible** fluid using index notation.

6. What is the Weissenberg number? Name a phenomenon discussed in class where it would play a key role.

7. What is the representation of the deviatoric stress τ_{ij} in terms of the velocity for an incompressible Newtonian fluid? (Index notation, please!)

8. Provide two physical interpretations for $\rho \underset{\sim}{u}$.

9. In the Navier-Stokes equations, what term would represent viscous diffusion of x-momentum in the y-direction?

10. A sphere of radius 1mm with thermal diffusivity $\alpha = 10^{-3}$ cm^2/s is dropped into hot oil. About how long will it take for the temperature at the center of the sphere to equilibrate?