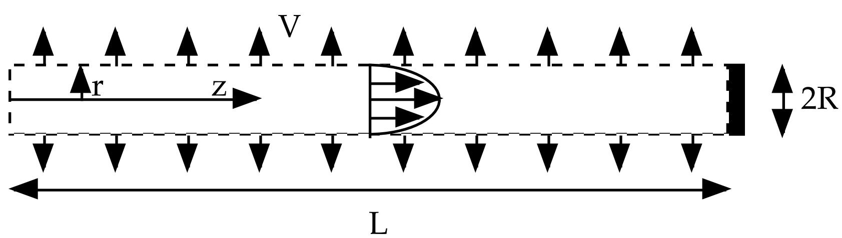
CHEG 355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Scaling/Creeping Flow: It is desired to use a hollow, porous fiber to perfuse nutrient into a tissue medium. The ID of the fiber is R, and its length is L, such that R/L <<1. While the radial perfusion velocity is normally a function of length due to the pressure drop along the length of the fiber, here we will make the simplifying assumption that the radial velocity at R is a constant V independent of z.



a. Render the governing equations dimensionless by appropriate scaling. By scaling the momentum equations, determine the simplified equations which govern the velocity and pressure in the limit R/L << 1 and VR/ν << 1.

b. Just like in lubrication problems, the axial pressure gradient is obtained from a mass balance (or volume balance, since the fluid is incompressible) over the tube. Using this, develop a differential equation for the pressure gradient.

c. Solve for the velocity distribution and pressure gradient. Hint: After you solve for v_z and the pressure gradient, you can get v_r from the continuity equation!

The following equations may be helpful:

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$
$$+\,\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

Problem 2. (20 pts) Dimensional Analysis: A classic problem in dimensional analysis (and an example used in class!) is the radius of a shockwave from an atom bomb as a function of time.

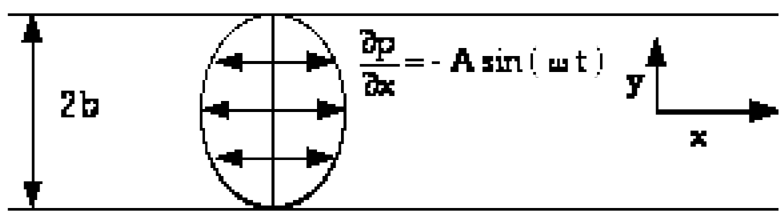
a. Given that the radius r of a shockwave in air depends only on the energy (yield) of the explosion E and the density of the gas prior to the explosion ρ_0 (the cold-gas approximation), use dimensional analysis to determine the radius of the shockwave as a function of time.

b. Eventually, the shockwave will expand to the point where the backpressure P_0 of the atmosphere it is expanding into will slow down the shockwave. Given that this critical radius depends only on the energy of the explosion E, and the atmospheric pressure P_0, use dimensional analysis to estimate the time at which the simple answer from part a breaks down.

Problem 3). (20 points) Inspectional Analysis: Consider the **unsteady** oscillatory flow in a channel of width 2 b depicted below. The fluid is incompressible, and the flow is unidirectional in the x-direction, with all that implies (hint: remember which of the inertial terms vanish!). We impose an oscillatory pressure gradient given by:

$$\frac{\partial p}{\partial x} = - A \sin(\omega t)$$

where A is the gradient amplitude and ω is the frequency of oscillation in time (the fluid sloshes back and forth in the x-direction).



a. Write down the momentum equation in the x-direction and show which terms are zero.

b. Render the equations dimensionless using $t^* = \omega t$ as the dimensionless time and U_C as an unknown velocity scale. Divide out by A to make the equations dimensionless.

c. The characteristic velocity U_C is determined by balancing the driving force in the problem (the pressure gradient) with either the inertial or viscous term. Recognizing this, determine this characteristic velocity for 1) high, and 2) low frequencies, and explicitly identify the single dimensionless group the problem depends on in either case (hint: it's the ratio of the momentum diffusion time to the period of oscillation).

Problem 4. (20 points) Short Answer

a. (10 pts) Briefly identify the physical mechanism described by each of the following terms:

1. $\mu \dfrac{\partial^2 u_x}{\partial y^2}$

2. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

3. $\dfrac{\partial u_i}{\partial x_i} = 0$

4. $\rho \dfrac{v_r v_\theta}{r}$

5. $\nabla^4 \psi = 0$

b. (10 pts) Multiple Choice:

1. In the study of insect flight, the boffins of Berkeley looked at the behavior of the 10cm scale model of a fruit fly in mineral oil. What dimensionless number were they trying to preserve?

A. Reynolds Number
B. Prandtl Number
C. Froude Number
D. Weissenberg Number

2). Which (if any) of the following can be discontinuous at a fluid-fluid interface?

A. Shear stress
B. Heat flux
C. Mass flux
D. Velocity

3). An ice skate slides on ice because:

A. Pressure concentration beneath the blade melts a thin layer of water
B. Frictional heating melts a thin layer of water
C. The coefficient of friction of frozen water is very low

4). Order the drag of the following objects under creeping flow conditions

A. A cube 2 cm on a side
B. A sphere 1.75 cm in radius
C. A sphere 2 cm in diameter

5). Estimate the Reynolds number of my big orange goldfish. Make any approximations (or guesses, if you haven't seen him) necessary, but give the basis of your estimate.