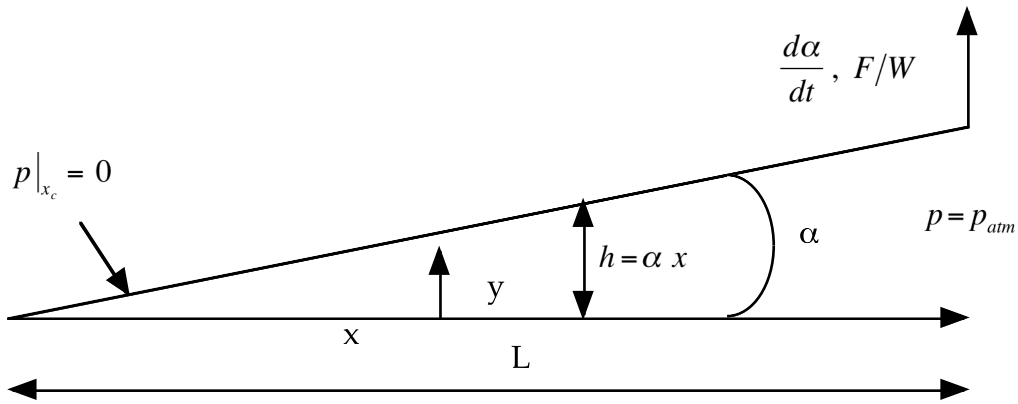
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Lubrication: Cavitation is the phenomenon which occurs when the absolute pressure in a liquid reaches the vapor pressure, and the liquid boils. For most liquids this is pretty close to zero. While usually seen in inertial flows, it is actually most easily visualized in viscous lubrication. Consider the geometry depicted below:



Two plates of length L in contact at one edge are separated by a very small angle α. For small angles the gap between the plates is given by h = α x, where x is the distance from the vertex. The plates are pulled apart at some rate dα/dt.

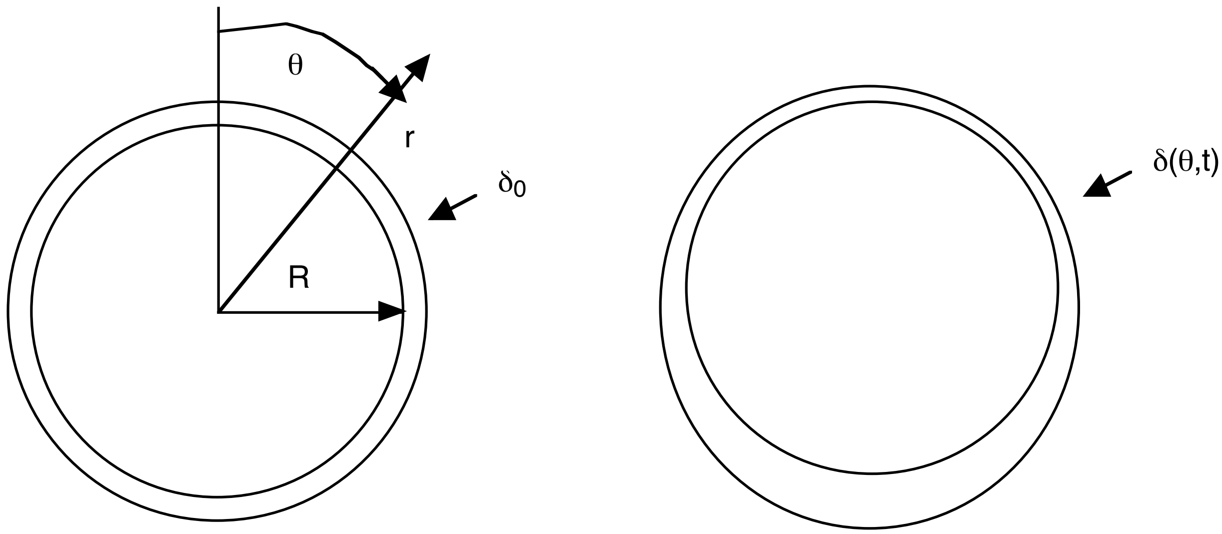
a. Using lubrication analysis, determine the location x_c where the pressure falls to zero absolute (e.g., a gauge pressure of - p_{atm}) and the fluid cavitates.

b. Develop an integral expression for the force per unit width into the paper F/W (exerted at the outer edge) necessary to pry the plates apart for some angular velocity dα/dt. (Hint: Think of torque balances!)

c. What happens to this force when the fluid cavitates? (don't solve for it – just be *briefly* qualitative)

Problem 2). (20 points) Drainage Flows: Earlier this semester your classmates demonstrated the drainage of a viscous film from an apple. Here we examine the simpler problem of drainage from a cylinder of radius R coated with a layer of fluid of density ρ and viscosity μ, and with initial thickness $\delta_0 << R$. We wish to determine the evolution of the thickness of the layer as a function of θ and t.

a. Redraw your coordinates for some value of θ in the flat earth limit (e.g., Cartesian coordinates!) and solve for the velocity profile in the draining film. Remember that g is now a function of θ! This should otherwise be identical to the falling film problem we solved in class. Note that $\delta \neq \delta_0$ as the film drainage evolves, rather that is just its initial condition!



b. Recognizing that the time derivative of the film thickness is just the radial velocity (or y velocity in your "flat earth" coordinate system) evaluated at δ(θ,t), develop the equation that δ must obey.

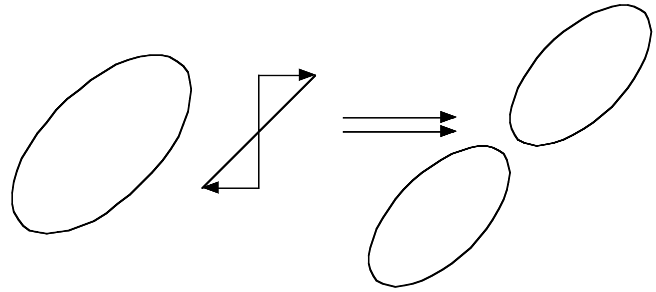
c. By scaling this equation, determine the characteristic drainage time t_c and render the problem dimensionless.

d. Solve for the time for the first drip to form (e.g., the solution blows up) at θ = π (the bottom of the cylinder).

I've provided the CE in cylindrical coordinates below, but it is not necessary for this problem because $\delta_0/R<<1$. It is much easier to do it in appropriate Cartesian coordinates, at least for the radial derivative!

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

Problem 3). (15 points) Dimensional Analysis: A popular way to make water-in-oil emulsions (basically suspensions of really tiny drops) is by subjecting a mixture of oil and water to high shear rates. The shear stresses in the oil tear apart the drops of water until they reach the desired size. It is observed that the final drop radius a is a function of the surface tension Γ, the shear rate $\dot{\gamma}$, the oil viscosity μ_f and the drop viscosity μ_d.



a. Form the dimensional matrix of the parameters governing this problem, and determine the number of dimensionless groups involved.

b. If $\Gamma = 4$ dyne/cm, $\mu_f = 70$cp, and the drop phase is water ***make an estimate*** of the oil phase shear rate required to achieve a drop radius of 5μm. Hint: it is the viscosity of the phase outside the drop which primarily controls the process!

c. A colleague responds to your result by saying "that's a really high shear rate – does inertia matter?" Quantitatively address this question.

d. One point extra credit: what is the name of the key dimensionless group?

Problem 4). (15 points) Short Answer: *Very* briefly answer the following:

1) What does Stokes' Law describe, and where is it valid?

2) What is its mathematical expression in terms of parameters (with the constant too!).

3) What does the Froude number represent?

4) Estimate the Reynolds number of a 0.5cm diameter minnow swimming at 10cm/s.

5) What equation does a harmonic function satisfy (name or mathematical expression)?

6) What does the material derivative represent?

7) What expression describes the volumetric flux normal to a surface, and what are its units?

8) What is the difference between strict and approximate dynamic similarity, giving one example?

9) What is a Jeffrey Orbit? (I demonstrated this in class)

10) What forms when the critical Taylor number is exceeded?

11) What equation governs the streamfunction for 2-D flow at zero Re?

12) What is a Stokeslet?

13) What is the difference between quasi-parallel and unidirectional flow?

14) What is the rate of strain tensor (index notation)?

15) Under what condition is the rate of strain tensor traceless?