PROBLEM SET 1 CBE30355 DUE 9/3/15

These first few problems should serve as a review for the vector calculus material you learned last semester and will use extensively this term.

1). Calculate the angle between the following pairs of vectors ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$):

a. (1,0,0), (0,0,2)

b. (1,1,1), (0,1,0)

c. (1,2,3), (3,2,1)

2). Calculate the following quantities (Note: X denotes the cross-product and · the dot product):

a. (1,0,1)·(1,0,-1)

b. (1,1,1)·(0,1,0)

c. (1,1,1)X(2,2,2)

3). For the scalar potential function $\phi = (x + y + z)^2$ and the velocity vector field $\mathbf{u} = (z, y, x^2)$ calculate the following vector quantities:

a. $\boldsymbol{\nabla}\phi$ ; $\boldsymbol{\nabla}\cdot\mathbf{u}$

b. $\nabla^2\phi = (\boldsymbol{\nabla}\cdot\boldsymbol{\nabla})\phi$ ; $\nabla^2\mathbf{u}$

c. $\boldsymbol{\nabla} X \mathbf{u}$

where the boldface operator $\boldsymbol{\nabla} = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

4. Prove that for an arbitrary vector $\mathbf{u}$:

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla} X \mathbf{u}) = 0$$

(In fluid mechanics, where $\mathbf{u}$ is the velocity, this is equivalent to saying that the vorticity [the curl of the velocity] is a solenoidal vector field [divergence free]. It is very useful in manipulating the equations of motion, particularly at high Reynolds numbers)

5. Two plates are separated by a distance of 1mm. A tangential force is applied to the upper plate (in the same manner as was described in class) and it begins to move, eventually reaching a steady velocity. Answer the following:

a. If the steady-state is achieved in about 5 seconds, estimate the kinematic viscosity of the fluid.

b. If the force is doubled, how does the time to steady-state change? (trick question…)

c. If the gap between the plates is halved, how does the time to steady-state change?

6. This is <u>completely</u> optional (and not for credit - solve only if you like puzzles): Prove the following vector identity for the arbitrary vector $\mathbb{u}$:

$$\mathbb{\nabla} \times (\mathbb{\nabla} \times \mathbb{u}) = \mathbb{\nabla} (\mathbb{\nabla} \cdot \mathbb{u}) - \nabla^2 \mathbb{u}$$

Hint: I usually solve this problem using index notation which is very useful for describing advanced transport problems. Detailed notes on index notation are available through the class website. We'll go over index notation in the first few review sessions.

In this problem set all vectors are in outlined boldface type while scalars are in regular type.