PROBLEM SET 3 CBE 30355 DUE 9/17/15

1). A classic "Honorable Mention" on the Darwin Awards website is the saga of "Lawn Chair Larry" who decided to go flying by attaching 42 helium filled 113 ft^3 weather balloons to his lawn chair (note: some sources list the balloons as 6ft, others as 4ft diameter. Which is more reasonable?). Instead of leveling off at around 30ft of altitude, he wound up at 16,000ft and actually was cited for violating LAX airspace. I'd like you to analyze this problem in hydrostatics and determine how many balloons Larry should have used - and if it is possible to control elevation with any precision. Use the "well-mixed atmosphere" assumption (adiabatic expansion) coupled with hydrostatics to get the variation of density and lift with altitude. Make any approximations you find necessary to get the answer in a reasonable amount of time (note that to be precise, you would have to know the elasticity of the balloons!) - although I'd certainly want more careful ones before -I- got in the lawnchair! One (of many) url's for the Lawn Chair Larry story is:

http://www.darwinawards.com/stupid/stupid1998-11.html

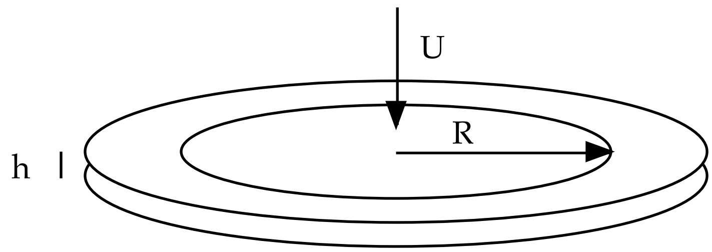
PS: A movie "Danny Deckchair" came out a while back, which is (very) loosely based on Larry's adventure. I've got the DVD, and it's pretty amusing. If you guys want to borrow it, let me know.

2). My office fish tank has about 45 gallons of water in it. In theory, the water is fresh – but ND tap water actually has quite a bit of salt in it. The water slowly evaporates at a rate of 1 gallon/week, and all the water evaporating is fresh (the salt stays in the tank). We want to see how the salt concentration changes with time under two different scenarios:

a. If I just top off the water every week with more tap water, how long will it be before the concentration increases by 50% (e.g., 1.5x the original value)? (this is really easy).

b. You are -supposed- to do weekly partial water changes (I tend to forget, alas). The idea is that you remove some water from the tank, and then replace it with more fresh tap water (after removing the chlorine, of course!). How much water should I remove/exchange each week so that the steady-state concentration is no more than 50% greater than the tap water concentration? What is the time for it to reach about 90% of this steady-state value (estimate only)? Note that the amount extracted each week is pretty small in comparison to the total volume of the tank, so you can approximate the process as continuous rather than discrete - this makes it much easier!

3). In our first lecture, I demonstrated what happens when a suspension is squeezed between two parallel-plates as is depicted below:



In this case the fluid of volume V is inserted between the plates, and the upper plate falls with a velocity U in the -z direction. The lower plate is fixed, so the gap width h is governed by the simple equation:

$$\frac{dh}{dt} = -U$$

a. As the plates move together, the fluid is squeezed out radially. If the initial separation is h_0, use conservation of mass to determine the radius R of the fluid between the plates as a function of time.

b. The radial velocity will be a function of radial position (it is zero in the center, for example). Using the continuity equation in cylindrical coordinates determine the average radial velocity (averaged over h) as a function of r and time.

c. In the limit of small h/R (such as was used in the demonstration), the velocity is dominated by the radial flow. This is the quasi-parallel approximation that always occurs in lubrication problems, such as we shall examine in detail later this term. For this geometry (and for a Newtonian fluid) the radial velocity is a parabola in z and proportional to a function of r and t (what you got in part b, actually). From the no-slip condition, it is also zero at both z=0 and z=h. Using this information, determine the radial velocity profile as a function of r, z, and t. Later on we'll use the momentum equations to determine pressure inside the fluid and the force required for this motion!

4). Index Notation: Using the concept of symmetry, isotropy and index notation, evaluate the following integrals over a **spherical** (e.g., isotropic) surface of radius a:

a. $\int_{r=a} x_1^2 x_2^2 \, dA$

b. $\int_{r=a} x_1^4 \, dA$

c. $\int_{r=a} x_2^2 \left(x_1^2 - x_2^2\right) dA$

d. $\int_{r=a} \left(x_1^2 + x_2^2 + x_3^2\right) dA$

e. $\int_{r=a} x_1 x_3^2 \, dA$

Hint: The integral $\int_{r=a} x_i \, x_j \, x_k \, x_l \; dA$ is a symmetric, isotropic fourth order tensor…