PROBLEM SET 5 CBE 30355 DUE 10/1/15

Read through Chapter 3 of BS&L.

In order to preserve the parallel between mass, momentum & energy transport BS&L defined the viscous stress tensor τ_{ij} to have a sign opposite of that defined in class and used in most other textbooks. If you keep this in mind, and recall that the sign of the stress tensor is completely arbitrary (as long as you are consistent!), it should not cause you any confusion.

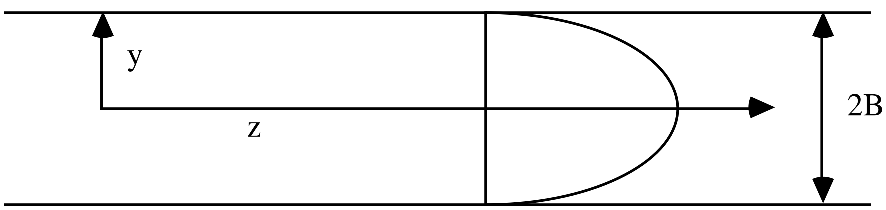
1). Consider a viscous fluid flowing in a laminar manner through a slit formed by two parallel walls a distance 2B apart as is depicted below.

a. Given that we have a uniform pressure gradient in the negative z direction (i.e., dp/dz equals a negative constant) and the fluid is massless with viscosity μ, calculate the flow rate using the Navier-Stokes equations.

b. What is the ratio of the centerline velocity to the average velocity?

c. What is the wall shear stress (force/area exerted by the fluid on the lower wall)?

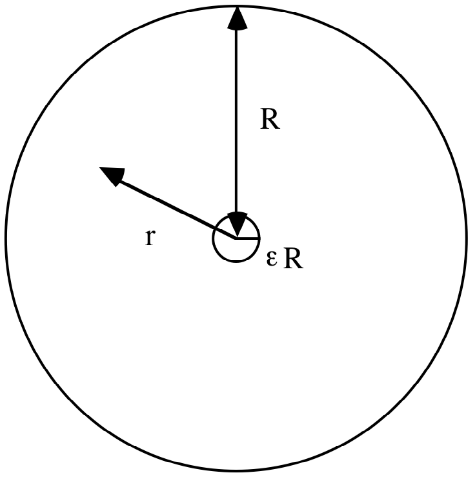
d. Now suppose we reorient the system so that gravity is in the positive z direction, the fluid has density ρ, and there is no pressure gradient (e.g., open to the atmosphere at both ends). What is the flow rate in this case?

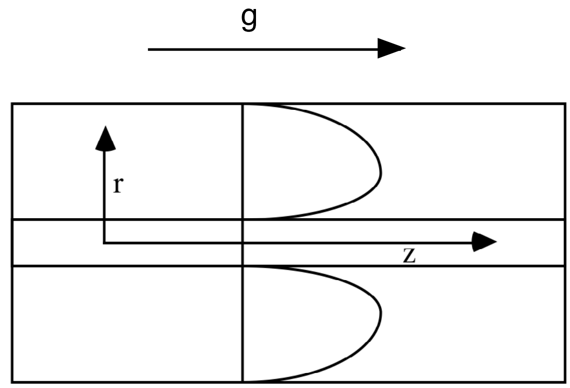


2. In class on Tuesday we will discuss the flow of a fluid through a pipe driven by a pressure gradient, as in problem 1 above. In this problem, consider a pipe of radius R with a small cylindrical wire of radius εR running axially down the center. If the remaining space in the tube is filled with a viscous fluid of density ρ and viscosity μ and there is no pressure gradient (the tube is open at both ends), calculate the resulting flow rate due to gravity as a function of ε and compare its magnitude to that when the wire is absent. The velocity profile needs to be obtained analytically, and isn't too bad, but the flow rate gets a bit messy. It *really really* helps to do this in dimensionless form! You may do the flow rate numerically (plotting it up as a function of ε) if you wish, or get it using Wolfram Alpha.

a. What is the flow rate as a function of ε? By how much is the flow rate reduced if ε = 0.02? Is this surprising?

b. Determine the force per unit length exerted by the fluid on the wire.

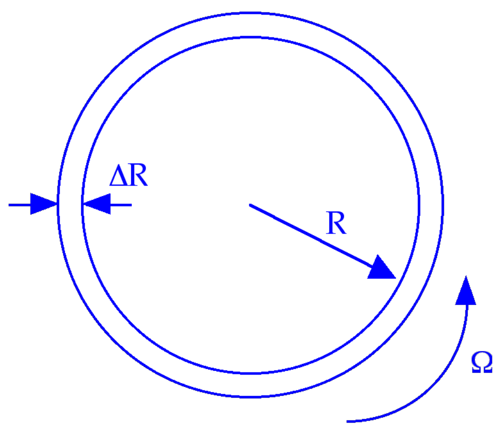




3. In class we derived a way to estimate the viscosity of a fluid using a Couette viscometer by neglecting the curvature effects (e.g., if the gap width is ΔR and the inner radius is R, we required $\Delta R/R << 1$).

a. Using cylindrical coordinates, derive the exact relationship between torque and rotation rate for arbitrary $\Delta R/R$ and graphically compare (e.g., use matlab to plot them up) the two results for $\Delta R/R$ in the range $0 < \Delta R/R < 1$. Note that if you get desperate, this problem is worked out completely in BS&L...

b. In my rheometer down in A68, my Couette device has a bob (inner cylinder) radius of 2.375 cm and a gap width of 1.261 mm. How large is the error in the calculated viscosity if we use the approximate formula for this instrument?



4. In the last homework you determined the complete *mobility tensor* for a falling body of revolution in a viscous fluid from two simple experiments: the measured velocity where the director was perpendicular to gravity, and where it was parallel to gravity. In class Mike measured the time for a washer to fall a set distance through Karo syrup as 13s if flat side on, and 11s if edge on. Using this information and the magic of index notation, we can predict the trajectory of the washer when dropped at an angle.

a. If the director p_i of the axis of revolution is given by:

$$p_i = \delta_{i1} \cos(\theta) + \delta_{i2} \sin(\theta)$$

where θ is the angle relative to vertical (e.g., gravity), plot up the lateral displacement of the washer after falling 10cm as a function of θ from 0 to $\pi/2$.

b. If the random error in each of Mike's two measurements was 0.2s, what is the 95% confidence interval of the lateral displacement for $\theta = \pi/4$?