PROBLEM SET 8 CBE 30355 DUE 11/5/15

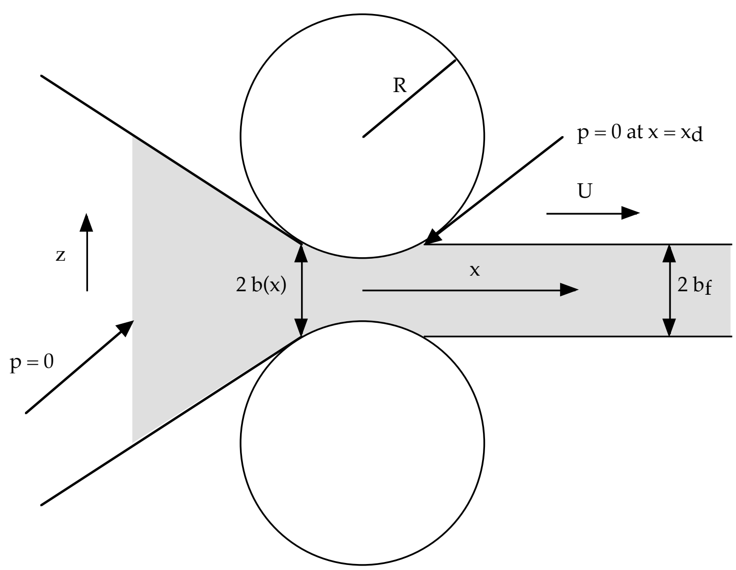
1). Consider the geometry below. Two sheets are drawn with a velocity U between two rollers of radius R whose surfaces are separated by a distance 2 b_0. The idea is to fill the space between the sheets with a viscous fluid of viscosity μ, and all the action takes place in a lubrication layer between the rollers, at least in the limit $b_0/R << 1$. Here we analyze this problem.

a. By scaling the flow equations in the gap (use Cartesian coordinates!), show how the force/width on the rollers F/L scales with the separation distance between the rollers and the other parameters of the problem.

b. The sheet sandwich detaches from the rollers downstream at a point x_d and final gap width 2 b_f > 2 b_0 when the pressure again returns to zero. Develop an implicit integral relation for the ratio b_f/b_0, and show that it doesn't depend on any of the other parameters of the problem.

c. Develop an integral relationship for the dimensionless pressure and force and evaluate it numerically.

Hint: the gap geometry in the lubrication limit is given by $b = b_0 + \frac{1}{2}\frac{x^2}{R}$ where x is the distance along the gap from the point of minimum separation. By continuity, the flow rate per extension into the paper (Q/W) through any vertical plane -must- be the same, and must equal the exit flow rate U*2b_f.



2. The sliding block:

a. For the sliding block considered in class with dimensions L = 20cm, W = 30cm, and d_2-d_1 = 0.01cm (e.g., a fixed inclination angle), determine the velocity U at which the block can support a weight of 100kg while maintaining a minimum separation d_1 = 0.01cm. Take the fluid viscosity to be 0.5 poise.

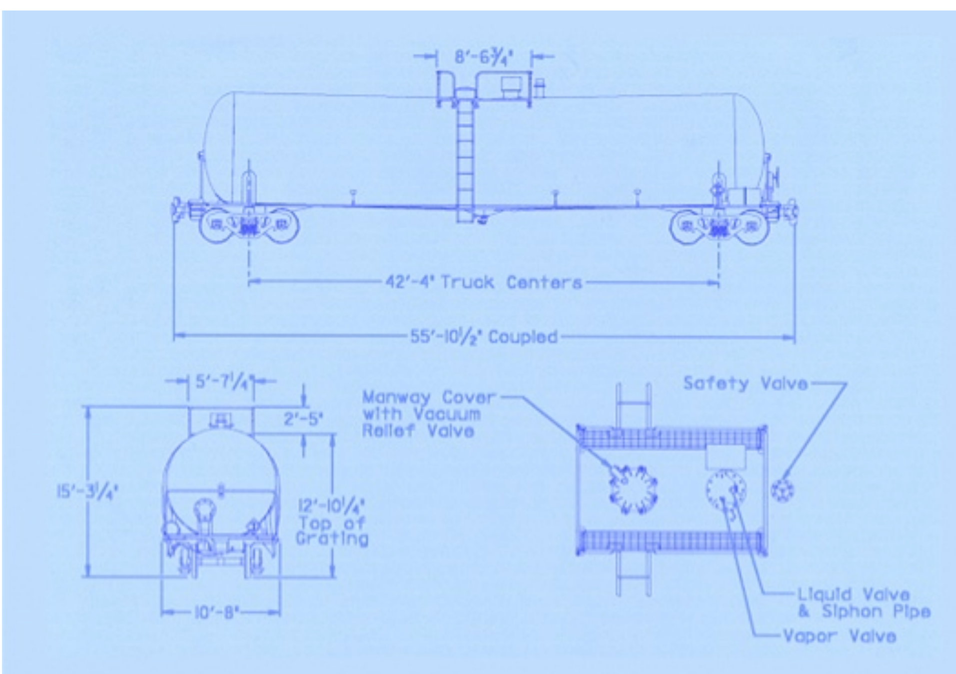
b. Calculate the drag on the block when it is fully lubricated at the conditions specified above (e.g., the lubrication equations apply!) and compare this to the drag when the lubrication fails (e.g., the drag is solely due to friction). Assume a coefficient of friction of 0.11, a typical value. Hint: it is *much* easier to determine the lubrication drag by calculating the shear stress on the plate – if you evaluate it on the block, you would also have to consider the contribution of pressure forces due to the slight incline angle of the block. Doing it right yields the same result either way, though.

3. Dimensional Analysis: Modeling air entrainment in a draining tank. The use of rail cars to ship heavy crude down from Canada or from the fields of North Dakota has exploded (sometimes literally, alas) in recent years. As a consequence of safety requirements, tank cars are being redesigned. You are assigned the task of performing a scale model test to determine the maximum drainage rate.

Consider the liquid tanker rail car depicted below. The tanker contains heavy crude with viscosity of 10 poise and density of 0.922 g/cm^3. As the tank drains a vortex will form and eventually air will be entrained into the drainpipe - something we want to avoid. We want to determine the allowable operating conditions for our large tank by studying the behavior of a model system (geometrically similar), but using water as the working fluid.

a. For strict dynamic similarity, what should be the scale down ratio of the scale model?

b. How does the draw off rate (volumetric flow rate) scale between the model and full size tanker?



4. Thermodynamics and Scaling Analysis: In this problem we estimate the temperature and fluid velocity in a chimney as a function of various physical parameters. The chimney is of cross-sectional area A and height H, thus the total volume of air in the chimney is AH. We have a source of energy at the bottom given by Q (assumed to be distributed over the whole area). The energy, of course, raises the temperature of the air, causing it to expand. Using the principles of conservation of energy and momentum, and neglecting all frictional and heat losses, estimate the velocity and temperature of the air in the chimney. Hint: You are going to have to remember some of your Thermodynamics from last term to get this one!