1. Flow inside a 90° trough. Consider the flow depicted below. A trough with an internal angle of 90° (e.g., the walls are at $\theta = \pm\theta_0 = \pm\pi/4$) is leaking out through a slit in the center with flow rate per unit extension into the paper Q/W. We want to solve for the velocity distribution. If we can ignore the inertial terms (Re << 1), the flow is governed by the Biharmonic Equation.

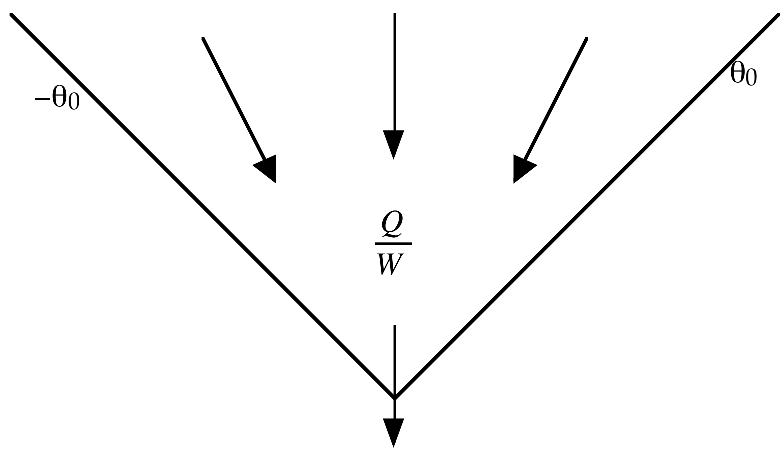
a. Write down the boundary conditions which govern the streamfunction for this problem.

b. We anticipate that the streamfunction will be of the form $\psi = \frac{Q}{W} r^{\lambda} f_{\lambda}(\theta)$. From the boundary conditions, what must λ be?

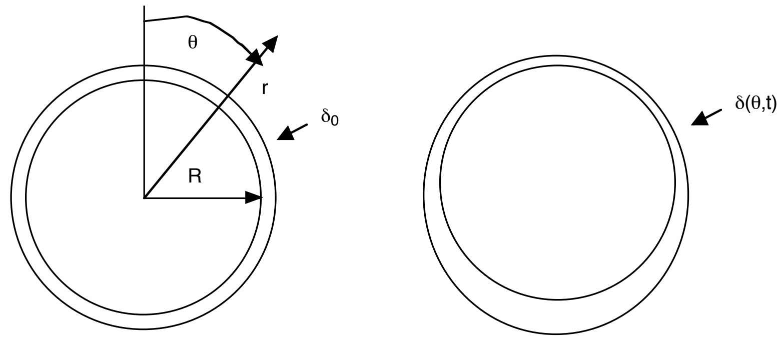
c. Solve for $f_{\lambda}(\theta)$. It is helpful to make maximum use of symmetry!

d. What is the radial velocity as a function of r and θ? Is there any flow in the θ direction?

Hint: The four homogeneous solutions for f(θ) are 1, θ, sin(2θ), and cos(2θ).



2. Film Drainage Flows: Earlier this semester your classmates demonstrated the drainage of a viscous film from an apple. Here we examine the simpler problem of drainage from a cylinder of radius R coated with a layer of fluid of density ρ and viscosity μ, and with initial thickness δ_0<<R. We wish to determine the evolution of the thickness of the layer as a function of θ and t.



a. Redraw your coordinates for some value of θ in the flat earth limit (e.g., Cartesian coordinates!) and solve for the velocity profile in the draining film. Remember that g is now a function of θ! This should otherwise be identical to the falling film problem we solved in class. Note that $\delta \neq \delta_0$ as the film drainage evolves, rather that is just its initial condition!

b. Recognizing that the time derivative of the film thickness is just the radial velocity (or y velocity in your "flat earth" coordinate system) evaluated at δ(θ,t), use the continuity equation to determine the timescale of the drainage problem t_c.

c. Now integrate the continuity equation to obtain the dimensionless partial differential equation that the film thickness must obey.

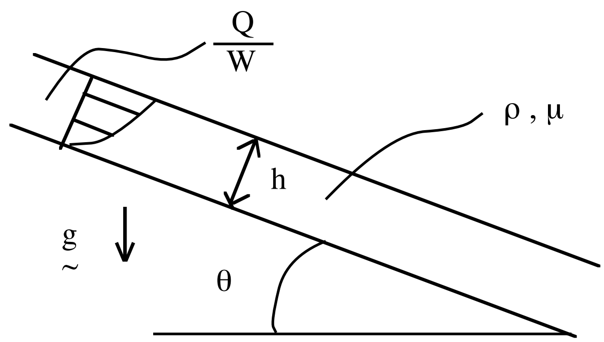
d. Solve for the time for the first drip to form (e.g., the solution blows up) at $\theta = \pi$ (the bottom of the cylinder).

3. Dimensional Analysis of Flow Down an Inclined Plane: Consider the inclined plane depicted below. The plane is inclined by an angle θ from the horizontal, and a fluid (viscosity μ, density ρ) is flowing down the plane with flow rate Q/W per unit width of the plane (normal to the plane of the paper - this is a two-dimensional problem). We wish to determine the thickness of this fluid layer as a function of the parameters of the problem using dimensional analysis.

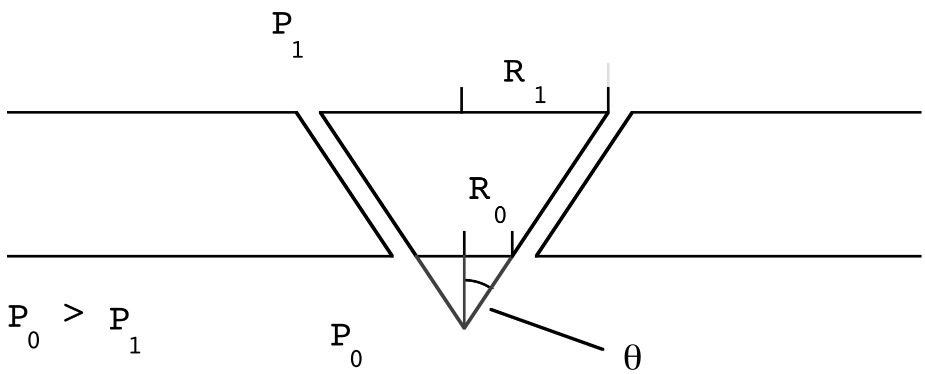
a. Form a dimensional matrix and prove that the problem involves just three dimensionless groups. Determine three independent dimensionless groups (Hint: one will be the Reynolds number).

b. At low Reynolds numbers we anticipate that the flow rate will be proportional to gravity. Use this to strengthen the result of the dimensional analysis, and determine the relationship between the flow rate and fluid thickness to within some unknown function of the angle of inclination. The thickness is proportional to what power of Q/W?

c. At very high Reynolds numbers we expect the flow to be fully turbulent and to no longer depend on the viscosity. Use this to determine a new relationship between the thickness and the flow rate in the high Re limit.



4. A conical cork is used to control the flow of air through a conical hole as is depicted below. For what values of R_1 and R_0 will the plug be blown out of the hole? The flow is considered to be ideal and inviscid, and the cork is massless. You should find that the force on the cork is independent of both the conical angle θ and the width of the gap surrounding the cork.



Hint: Assuming parallel flow in the gap, use Bernoulli's equation and continuity to determine the velocity in the gap. To simplify the algebra, use a coordinate system s defined from the imaginary vertex of the cone, calculate P(s), and integrate over s from s_0 to s_1 where $s_0 = R_0/\sin\theta$ and $s_1 = R_1/\sin\theta$.