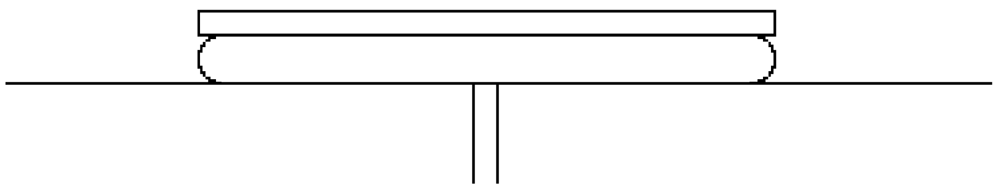
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Note: Problems are not equally weighted!

Problem 1). (30 points) Scaling Theory: High Reynolds Numbers to Low! Consider a disk of radius R located a distance H above a plane as depicted below. Fluid of viscosity μ and density ρ is pumped into the center of this geometry at a rate Q (volume/time), and flows outward radially. It is observed that at low flow rates the disk is blown away from the plane, while at high flow rates it is sucked downwards. Here we analyze this problem in the limit $H/R << 1$. You may take the flow to be only in the radial direction!



a. Using a mass balance, determine the characteristic magnitude of the radial velocity and how it depends on r.

b. Via scaling analysis of the equations of motion and a force balance on the plate, determine how the force scales with the parameters of the problem at high flow rates. How large does the flow rate have to be for this asymptotic limit to be valid (e.g., how does it depend on R, H, ρ, μ, etc.)?

c. Determine via scaling analysis how the force scales with the parameters of the problem at low flow rates. How low does the flow rate have to be for this asymptotic limit to be valid?

d. If we allow the gap width to change, it will eventually seek a separation where the net force is zero. How does this depend on the flow rate?

The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 2). (20 points) Lubrication: Detachment of a Sphere from a Plane: Consider the geometry depicted below. A sphere of radius a is separated from a plane by a distance h_0 such that $h_0/a << 1$. In the narrow gap limit, the gap width h is given approximately by:

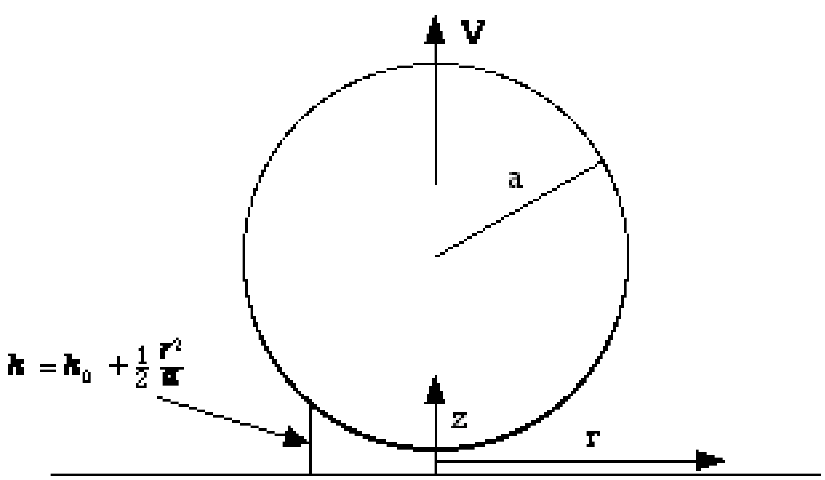
$$h = h_0 + \frac{1}{2}\frac{r^2}{a}$$

where r is the radial distance from the minimum separation (cylindrical coordinates).

a. Determine the equations which govern the pressure distribution and force on the sphere in the lubrication limit.

b. Using scaling analysis, determine the force necessary for the sphere to have a vertical velocity V for small h_0/a to within some unknown numerical value. Note that the appropriate length scale for r is -not- the radius of the sphere!

c. Solve for the numerical value. You can leave the dimensionless force and pressure in terms of integrals if you wish, but they are actually easy to evaluate if you do it right...



Problem 3). (10 pts) Dimensional Analysis: You are assigned the task of estimating the drag on a 300 meter long aircraft carrier moving at 20 m/s (roughly 40 knots, or 40 nautical miles per hour). You choose to do this using a towing tank with a 1 meter long model.

a. The dimensionless drag depends on two key dimensionless parameters: what are they, and what is their approximate magnitude for the full-sized ship? (Hint: If you can't remember them, you can easily get them by appropriate scaling of the Navier-Stokes equations!)

b. Using the concept of approximate (rather than strict) dynamic similarity, determine the velocity at which the model should be towed to best approximate the flow pattern of the full-sized carrier. What are the values of the important dimensionless parameters for the model and what is the ratio of the drag on the carrier to that measured for the model?

Problem 4). (20 points) Short Answer / Multiple Choice:

a. (10 pts) Briefly identify the physical mechanism described by each of the following terms **and a problem where it would play a role**:

1. $\mu \dfrac{\partial^2 v_\theta}{\partial z^2}$
2. $\dfrac{D\rho}{Dt}$
3. $\nabla^4 \psi = 0$
4. $\tilde{u} = -\tilde{\nabla} \phi$
5. $p + \frac{1}{2}\rho u^2 + \rho g h = Cst$

b. (10 pts) Multiple Choice:

1). Which (if any) of the following can be discontinuous at a fluid-fluid interface?

A. Shear rate
B. Heat flux
C. Mass flux
D. Velocity

2). Order the drag of the following objects under creeping flow (Re << 1) conditions

A. A cube 2 cm on a side
B. A sphere 1.75 cm in radius
C. A sphere 2 cm in diameter

3). Estimate the Reynolds number of my big orange goldfish. Make any approximations (or guesses, if you haven't seen him) necessary, but give the basis of your estimate.

4). A paramecium is a single celled animal around 200μm in size that tends to swim around in circles with a velocity of 1mm/s. Estimate its Reynolds number.

5). In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 10m boat of this type?