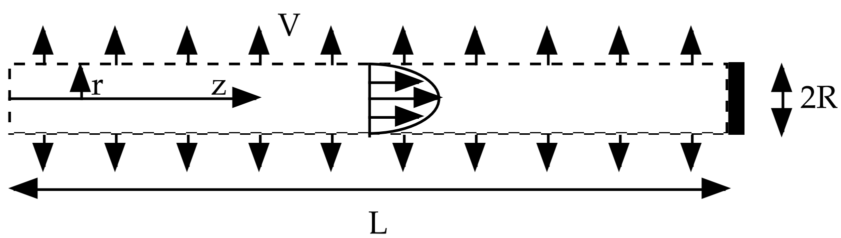
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Scaling/Quasi-Parallel Flow: It is desired to use a hollow, porous fiber to perfuse nutrient into a tissue medium. The ID of the fiber is R, and its length is L, such that R/L <<1. While the radial perfusion velocity is normally a function of length due to the pressure drop along the length of the fiber, here we will make the simplifying assumption that the radial velocity at R is a constant V independent of z. The end of the fiber is plugged, so that all fluid entering at z = 0 exits through the porous walls (useful for doing a mass balance!).



a. Render the governing equations dimensionless by appropriate scaling. By scaling the momentum and continuity equations, determine the simplified dimensionless equations and boundary conditions which govern the axial and radial velocity and pressure in the limit $R/L << 1$ and $VR/\nu << 1$.

b. Using conservation of mass (or volume, since the fluid is incompressible), determine the average axial velocity as a function of z.

c. Using your knowledge of flow through a pipe, solve for the velocity distribution in both the r and z directions and the pressure gradient. Hint: After you solve for v_z and the pressure gradient, you can get v_r from the continuity equation! You can *really* short-circuit the calculations if you recall the ratio of centerline to average axial velocity for this geometry!

The following equations will be helpful - just remember that most of the terms are zero!

$$\frac{1}{r}\frac{\partial\left(r\,v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

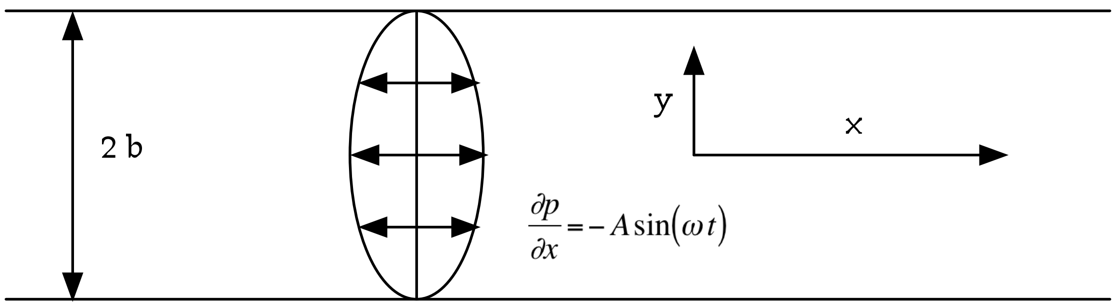
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

Problem 2). (20 points) Scaling / Unidirectional Flows: Consider the **unsteady** oscillatory flow in a channel of width 2 b depicted below. The fluid is incompressible, and the flow is unidirectional in the x-direction, with all that implies (**Hint**: Remember which of the inertial terms vanish!). We impose an oscillatory pressure gradient given by:

$$\frac{\partial p}{\partial x} = -A\sin(\omega t)$$

where A is the gradient amplitude and ω is the frequency of oscillation in time (the fluid sloshes back and forth in the x-direction).



a. Write down the momentum equation in the x-direction and show which terms are zero. Render the equations dimensionless using $t^* = \omega t$ as the dimensionless time and U_C as an unknown velocity scale. Divide out by A to make the equations dimensionless. The characteristic velocity U_C is determined by balancing the driving force in the problem (the pressure gradient) with either the inertial or viscous term. Recognizing this, determine this characteristic velocity for 1) high, and 2) low frequencies, and explicitly identify the single dimensionless group the problem depends on.

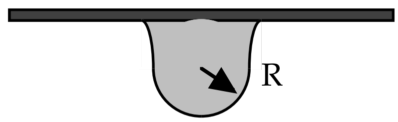
b. Solve for the velocity profile in the high frequency limit. What happens to the boundary conditions?

c. Solve for the velocity profile in the low frequency limit.

Problem 3). (10 pts) Dimensional Analysis: Under some conditions (slow drips seeping from the underside of a porous plate) the radius R of a drop detaching from the surface is the result of a balance between the surface tension Γ (units Force/Length) and gravitational forces.

a. Form the dimensional matrix of the parameters governing this problem, and determine the number of dimensionless groups involved.

b. Using the results of part (a) give a **rough estimate** of the radius R of a drop of water ($\Gamma = 70$ dynes/cm) dripping off of a plate and calculate its volume.



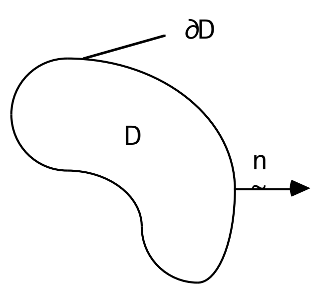
Problem 4. (10 pts) Consider the arbitrary fluid element depicted below. A fluid with thermal energy per unit volume $\rho\, C_p\, T$ (where T is the temperature and C_p is the heat capacity per unit mass) is flowing through the element with velocity $\underset{\sim}{u}$. In addition to convective heat transfer, we also have the conductive (diffusive) heat transfer flux, given by Fourier's Law of heat conduction:

$$\underset{\sim}{q} = -k \underset{\sim}{\nabla} T$$

which is analogous to the shear stress from Newton's law of viscosity. Here k is the thermal conductivity. There is also a source of thermal energy (heat generation) per unit volume S due to chemical reactions or other sources. Both the velocity $\underset{\sim}{u}$ and the temperature T may be functions of position and time, but physical properties such as k, ρ, and C_p are taken to be constant.

a. Given the statement that thermal energy is conserved, derive an integral equation governing the thermal energy in the arbitrary control volume depicted below. Don't forget the accumulation term!

b. Using the results of part a, derive the microscopic equation governing energy transport in a flowing liquid analogous to the Navier-Stokes equations. You will use this equation *a lot* next term!



Problem 5). (20 points) Short Answer / Multiple Choice:

For the five expressions below, briefly identify the corresponding physical mechanism (or equation name) **and a problem where it would play a role**:

1. $\rho \dfrac{D\underset{\sim}{u}}{Dt} = -\underset{\sim}{\nabla} P$

2. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

3. $\nabla^4 \psi = 0$

4. $\dfrac{D\phi}{Dt} \equiv \dfrac{\partial \phi}{\partial t} + \underset{\sim}{u} \bullet \underset{\sim}{\nabla} \phi$

5. $Q = \dfrac{-\pi}{8} \dfrac{\Delta P}{L} \dfrac{R^4}{\mu}$

6. Which (if any) of the following can be discontinuous at a fluid-fluid interface?
   A. Shear stress
   B. Heat flux
   C. Mass flux
   D. Velocity

7. Order the drag (from smallest to largest!) of the following objects under creeping flow (Re << 1) conditions
   A. A cube 2 cm on a side
   B. A sphere 2 cm in diameter
   C. A sphere 1.75 cm in radius

8. In senior lab yesterday a student tried to get the last bit of glycerin off of the walls of a graduated cylinder by shaking it up and down (e.g., an oscillatory axial motion). Using your knowledge of creeping flow, briefly explain why this did not increase the drainage rate. Three words will suffice if they're the right ones!

9. *Briefly* explain the difference between quasi-parallel and unidirectional flows, providing an example of each.

10. In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel ships) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 40m long ship of this type?