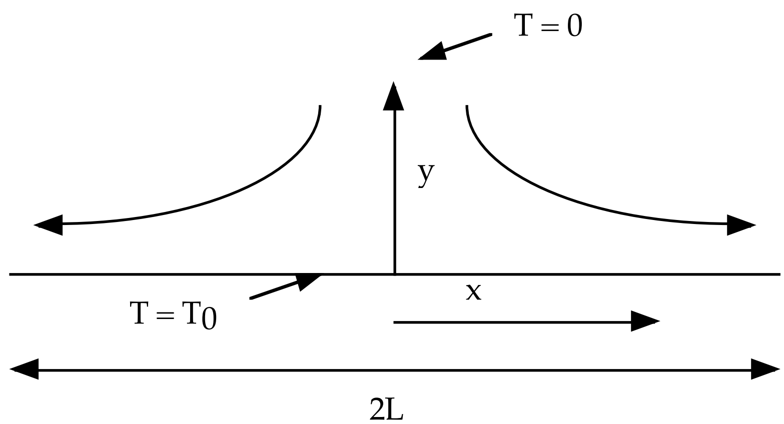
CBE 30355 Transport Phenomena I
Final Exam

December 12, 2012

Closed Books and Notes

Problem 1. (20 points) Scaling/Boundary Layers: Consider the heat transfer from a plate in the flow due to an impinging planar jet as depicted below. The fluid from the jet (usually produced by flow from a slit) has a temperature T = 0 (e.g., we subtract it off as a reference) impinges on a plate maintained at a temperature T_0. Your mission is to determine the total heat transfer from the plate per unit extension into the paper, Q/W.



$$\rho \hat{C}_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad ; \quad \boxed{Q/W = 2 \int_0^L -k \left. \frac{\partial T}{\partial y} \right|_{y=0} dx}$$

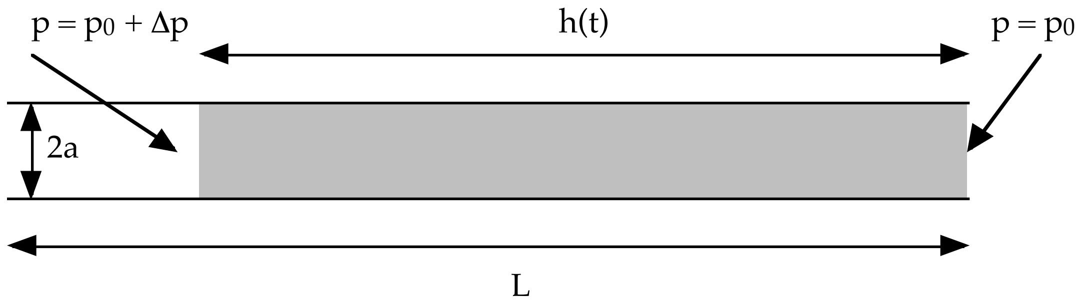
a. The velocity in the x-direction near the plane is given by u = λxy. Applying no-slip and continuity, what is the velocity v in the y-direction?

b. The heat transfer equations are given below. Scale the equations in the boundary layer limit to determine the boundary layer thickness δ. Scale the expression for the integrated heat flux to determine the total rate of heat loss from the plate to within some unknown O(1) constant (e.g., the dimensionless solution to the problem).

c. Derive the second order ODE which governs the problem, together with boundary conditions.

d. For a bonus point (e.g., do it if you have time left!), solve the differential equation and get the heat flux. Leave it in integral form.

Problem 2. (20 pts) Scaling/Unidirectional flows: Consider a horizontal straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below. The liquid is initially at rest. At time t=0 we start to blow the liquid out of the straw by applying a constant pressure differential Δp. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw in both the high and low Re limits - e.g., how long does it take for h to reach zero.



a. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

c. Solve the problem to obtain the dimensionless non-linear second order ODE which governs the evolution of the liquid slug length in this limit, together with initial conditions. This equation is trivial to solve numerically using matlab, of course, (the dimensionless blowout time comes out to be $(\pi/2)$^.5) but don't do it here!

d. Scale the equations for LOW Reynolds numbers, and determine the new (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

e. Solve for the actual drainage time in the low Reynolds number limit (e.g., solve the dimensionless equations obtained in part d to get the numerical value).

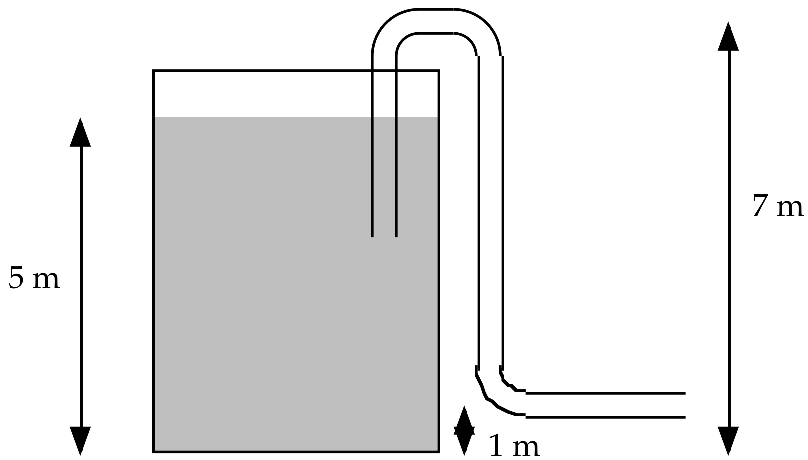
The following equations may be helpful:

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Problem 3). (20 points) A Simple Siphon. Here we determine the flow rate of a siphon used to drain the tank depicted below. Water fills a tank to the height shown, and the siphon consists of 16 meters of 5cm ID pipe (it is filled with water too!).



a). Neglecting all frictional losses, what is the flow rate? Give your answer in liters/s.

b). Modify your answer by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You will probably need to do a couple of iterations to get the friction factor right. It helps to start the iterative calculation of the velocity with a reasonable guess for f_f since you know the other parameters.

$$h_L = \frac{\langle u\rangle^2}{2g}\sum K + 4\, f_f\frac{L}{D}\frac{\langle u\rangle^2}{2g}$$

$$f_f = \frac{16}{Re}\ ;\ \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}}\ ;\ 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0\log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40\ ;\ \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.7

Problem 4). (30 points) Pump Curves / Additional Readings / Short Answer:

The first six questions refer to the pump curve on the next page:

1. It is desired to pump 24 liters/sec of water from a pond to an elevation of 10 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP80i recommended for the job?

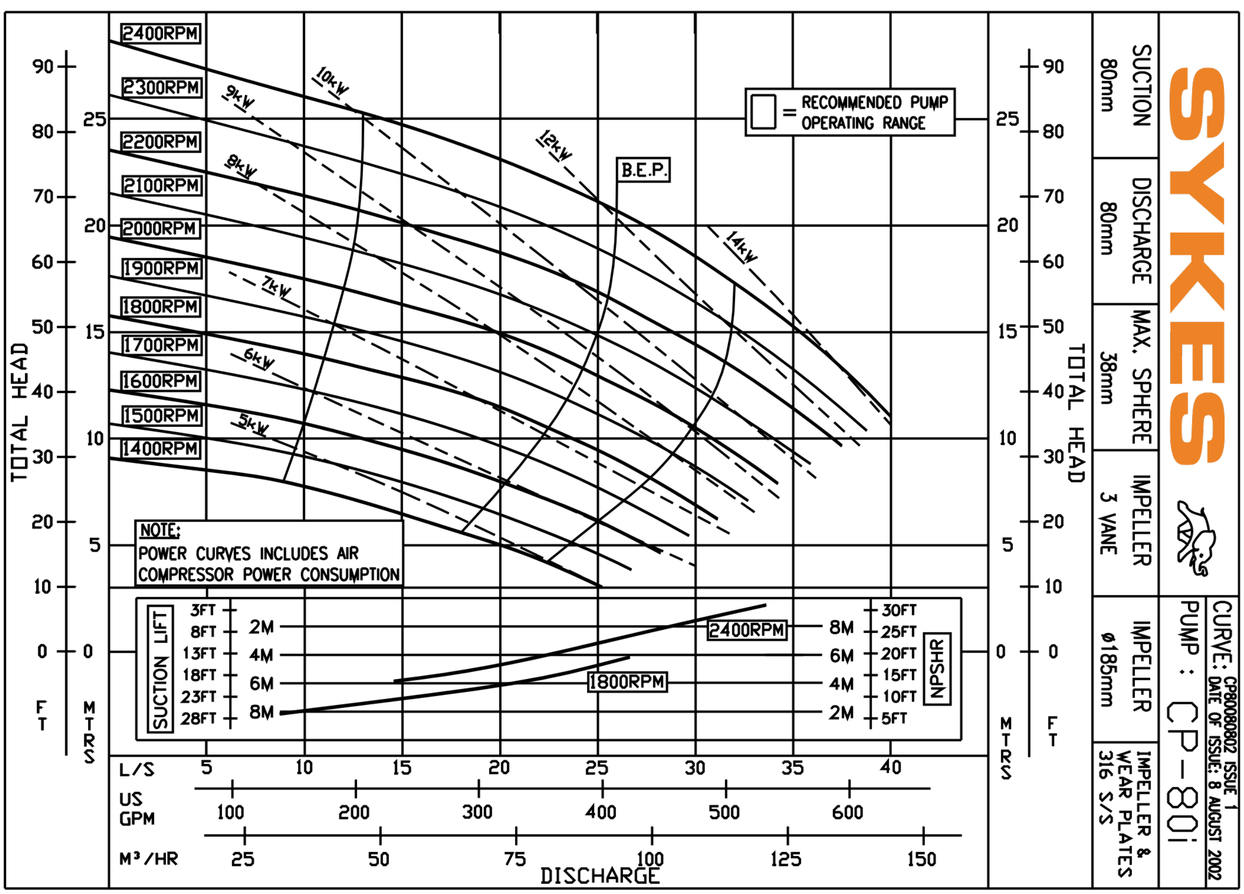
2. What is the RPM required to do the job?

3. What is the useful mechanical work done <u>by</u> the pump <u>on</u> the fluid?

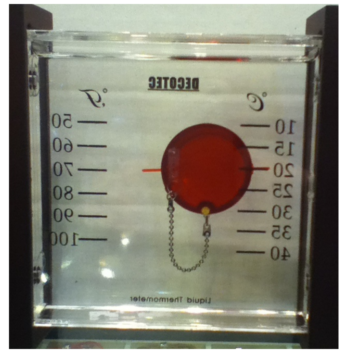
4. What is the efficiency of the pump at the operating conditions?

5. What vertical distance from the level of the pond can we put the pump at? (Again, neglect frictional losses)

6. If we are pumping hot water at 60^0C (vapor pressure of 150mmHg vs. that at 25^0C of 23.7mmHg), what is the maximum vertical distance? (OK, a hint: mercury has a density of 13.56 times that of water…)



7. At the Mole Hole (a gift store on the East Race) I saw an interesting variant on a Galileo's Thermometer. Front and back pictures of it are reproduced below. Briefly explain how it works (it is filled with a fluid, probably water).



8. Just as in the case of turbulent momentum transfer where we define a turbulent kinematic viscosity ν_t, so in the case of turbulent energy transfer we can define a turbulent thermal diffusivity α_t. The turbulent Prandtl number is the ratio of these quantities ($Pr_t = \nu_t \,/\, \alpha_t$). What is its approximate magnitude and why?

9. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

10. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

11. For a shear stress of 36 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

12. What problem is called Stokes' Paradox? How is it resolved?

13. What is D'Alembert's Paradox? How is it resolved?

14. Sketch and briefly explain the principle behind a venturi flow meter.

15. Explain the difference between quasi-parallel and unidirectional flows