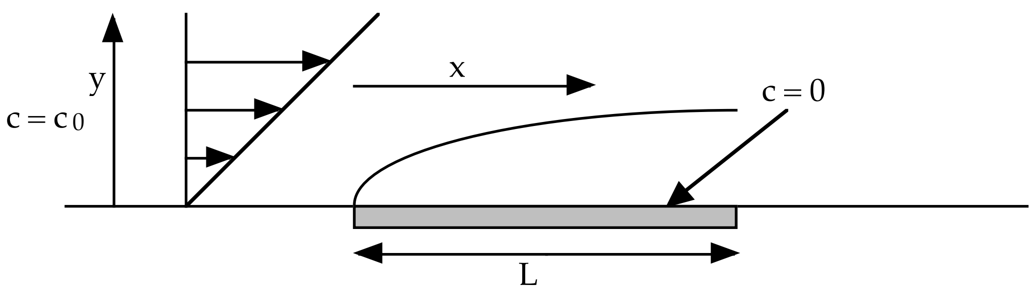
CBE 30355 Transport Phenomena I
Final Exam

December 19, 2013

Closed Books and Notes

Problem 1. (20 points) Scaling analysis of boundary layer flows. A popular method for measuring instantaneous wall shear stresses in turbulent flows is the use of an electrochemical probe. The fluid is doped with some reactant at concentration c_0 which decomposes electrochemically at an electrode flush with the wall. The reaction is essentially instantaneous, so the concentration of reactant at the electrode is zero, and the reaction rate is determined by the rate with which the stuff diffuses to the wall. By measuring the current, we can determine the reaction rate and hence the integrated mass flux to the electrode. The mass flux gets related to the shear stress because diffusive transport of mass is so slow relative to momentum transport that the mass transfer boundary layer essentially samples the linear shear flow right at the wall. We thus have the following problem:



$$\underset{\sim}{u} \cdot \underset{\sim}{\nabla} c = D \nabla^2 c \quad ; \quad \underset{\sim}{u} \sim \underset{\sim}{\hat{e}}_x \frac{\tau_w}{\mu} y$$

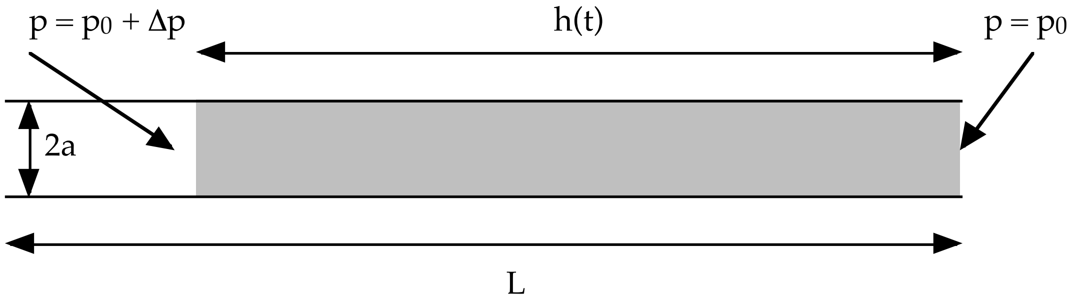
$$N\Big|_{y=0} = -D \frac{\partial c}{\partial y}\Big|_{y=0} \quad ; \quad I = W \int_0^L N\Big|_{y=0} dx$$

a. Using scaling analysis, determine how the current per unit width I/W scales with the parameters of the problem (e.g., τ_w, D (the diffusion coefficient), μ, c_0, and L).

b. Estimate the time resolution of such a probe (e.g., how long would it take for the signal to approach steady state in the presence of shear transients). Choose L = 1mm, D = 10^{-6} cm^2/s, τ_w = 100 dynes/cm^2, and the properties of water.

c. The current is only related to the shear stress if the boundary layer approximation holds. In general, we would like to make the probe as short as possible to get the best time resolution possible. What is the shortest length the probe can be?

Problem 2. (20 pts) Scaling/Unidirectional flows: Consider a horizontal straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below. The liquid is initially at rest. At time t=0 we start to blow the liquid out of the straw by applying a constant pressure differential Δp. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw in the high Re limit - e.g., how long does it take for h to reach zero.



a. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

c. Solve the problem to obtain the dimensionless non-linear second order ODE which governs the evolution of the liquid slug length in this limit, together with initial conditions. This equation is trivial to solve numerically using matlab, of course, (the dimensionless blowout time comes out to be a nice O(1) value of (π/2)^.5) but don't do it here!

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\boxed{\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z}$$

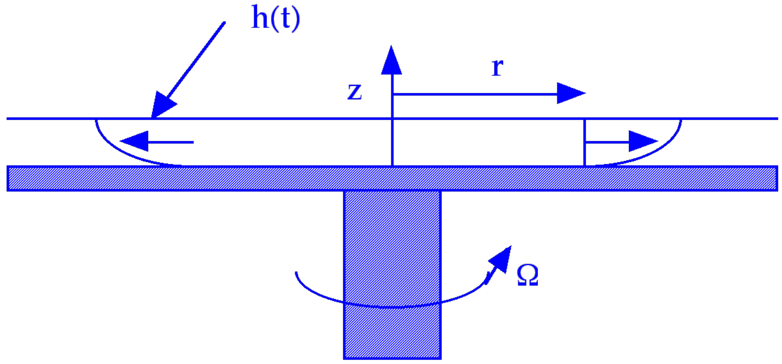
$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\left(r\,u_r\right)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+\rho g_r$$

Problem 3. (20 points) Scaling/Lubrication: A common way to produce a thin film on a surface is to spin coat it - take a disk of radius R, deposit some viscous liquid of kinematic viscosity ν on the surface, and then rotate it with some angular velocity Ω. Centrifugal forces cause the fluid layer to thin out over time, leaving a final thickness $h_f << R$ at the end of the process. Here we analyze this procedure.

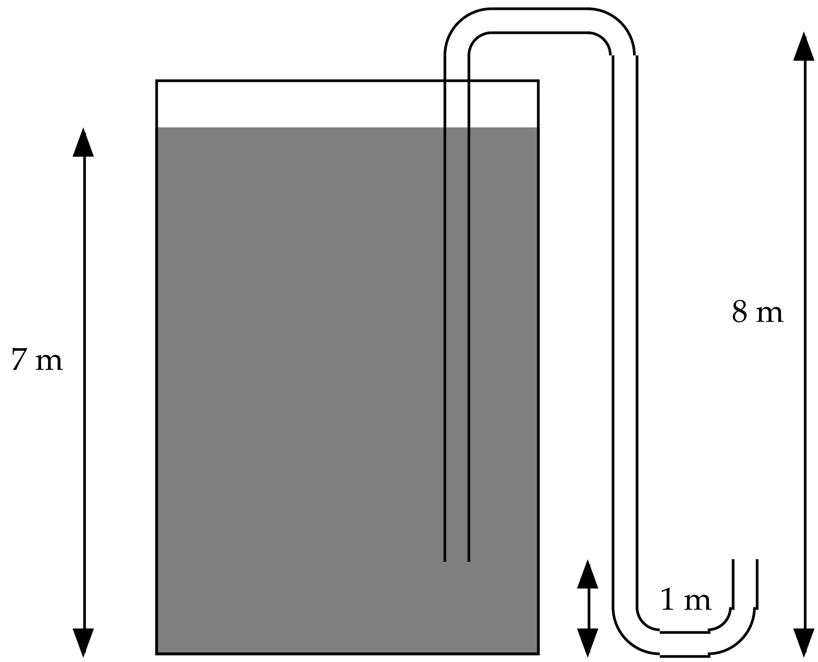
a). For *thin* fluid layers the rotational velocity is just $u_\theta = \Omega r$ over the entire film (e.g., it moves with the rotational velocity of the disk). Using this, determine the differential equations and boundary conditions which govern the radial and vertical velocity distribution in the film, and the equation for the change in film thickness over time. (Hint: How does the change in thickness with time relate to u_z at $z = h(t)$?) It is appropriate to use h_f, the final thickness, as the vertical length scale.

b). Using scaling analysis, determine how long the spin coating process takes as a function of the parameters of the problem to within the usual unknown O(1) constant.

c). Explicitly solve for the velocity distribution and the height as a function of time to get the O(1) constant. You may take the initial height to be H where $H/h_f >> 1$.



Problem 4. Here we determine the flow rate of a siphon used to drain the tank depicted below. Water fills a tank (open at the top to the atmosphere) to the height shown, and the siphon consists of 16 meters of 4cm ID pipe (it is filled with water too!).



a). Neglecting all frictional losses, what is the flow rate and what height (relative to the siphon outlet) would the spray from the siphon outlet reach? Give your answer in liters/s for the flow rate and meters for the spray height.

b). Modify your answer to the flow rate and spray height by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You will probably need to do a couple of iterations to get the friction factor right - don't forget the domain of validity of the correlations! It helps to start the iterative calculation of the velocity with a reasonable guess for f_f since you know the other parameters.

$$h_L = \frac{\langle u \rangle^2}{2g} \Sigma K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.7

Problem 5). (20 points) Pump Curves / Additional Readings / Short Answer:

The first five questions refer to the pump curve below:

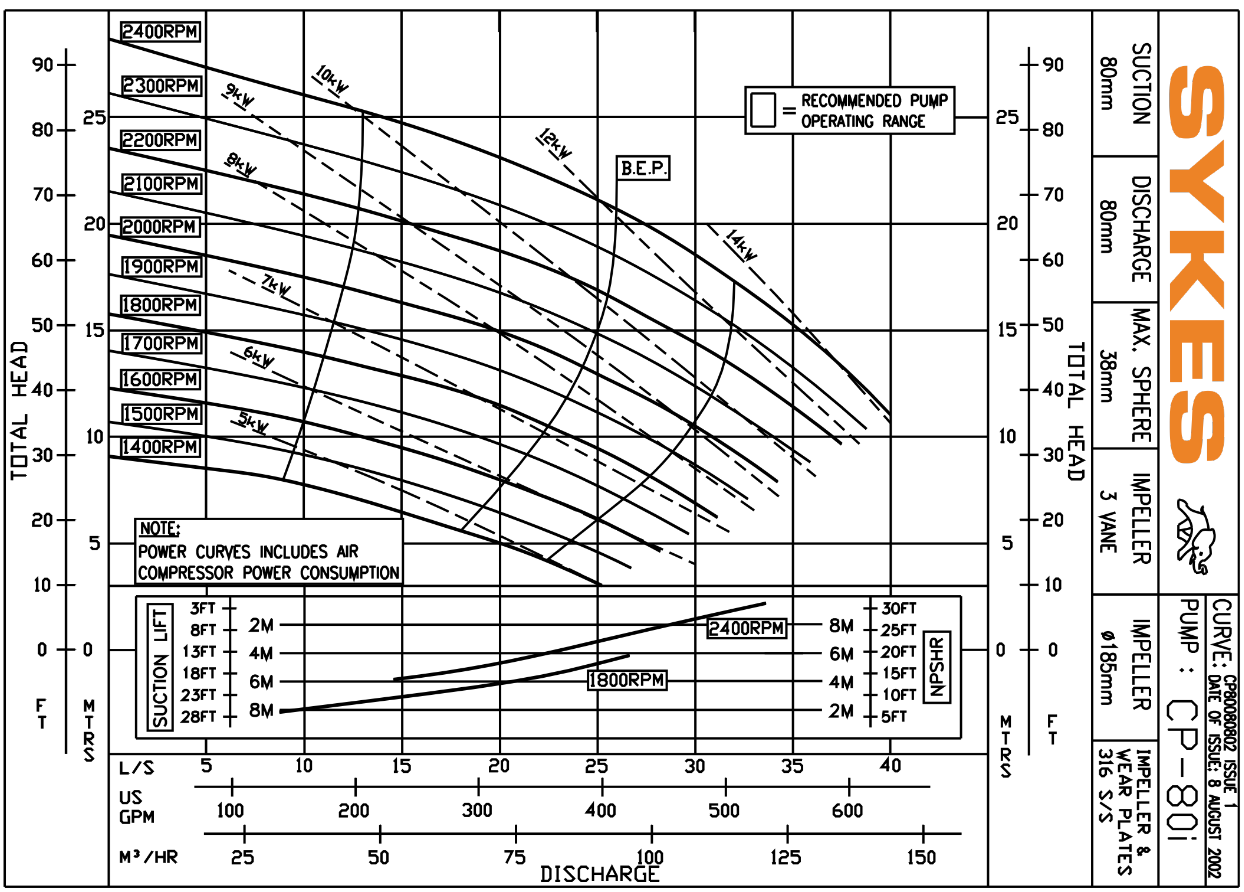
1. It is desired to pump 33 liters/sec of water from a pond to an elevation of 15 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP80i recommended for the job?

2. What is the RPM required to do the job?

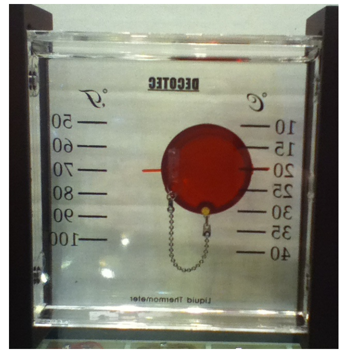
3. What is the efficiency of the pump at the operating conditions?

4. What vertical distance relative to the level of the pond can we put the pump at? (Again, neglect frictional losses)

5. If we are pumping hot water at 60^0C (vapor pressure of 150mmHg vs. that at 25^0C of 23.7mmHg), and the tank is in Denver (630mmHg), what is the vertical distance? (OK, a hint: mercury has a density of 13.56 times that of water…)



6. At the Mole Hole (a gift store on the East Race) I saw an interesting variant on a Galileo's Thermometer. Front and back pictures of it are reproduced below. It is filled with a fluid, probably water, and one end of a chain is attached to the wall, and the other to the floating disk (you can see this more clearly from the back side picture). Briefly explain how the chain makes the thermometer work.



7. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

8. Why do frisbees have ridges? Briefly, please!

9. For a shear stress of 25 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

10. Explain the key difference between quasi-parallel (e.g., what you get for lubrication flows or the Blasius problem) and unidirectional flows.