CBE 30355   TRANSPORT PHENOMENA I   TTh 11:00

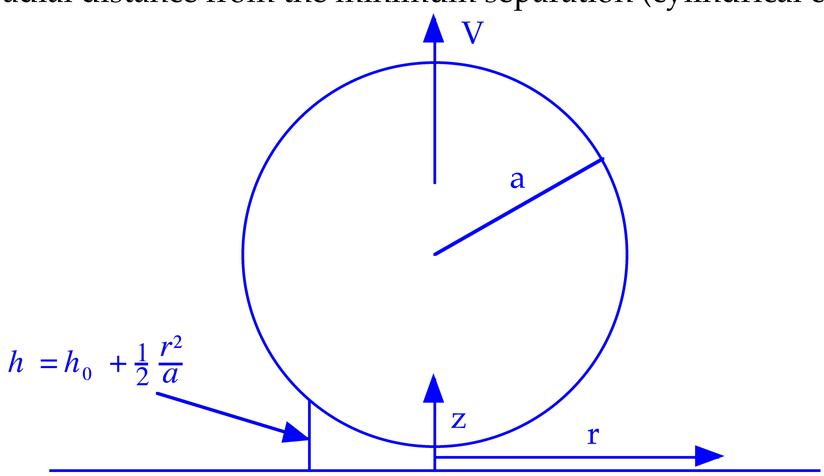
Second Hour Exam

Closed Books and Notes

Problem 1). (15 points) Lubrication: Detachment of a Sphere from a Plane: Consider the geometry depicted below. A sphere of radius a is separated from a plane by a distance h_0 such that $h_0/a << 1$. In the narrow gap limit, the gap width h is given approximately by:

$$h = h_0 + \frac{1}{2}\frac{r^2}{a}$$

where r is the radial distance from the minimum separation (cylindrical coordinates).



a. Determine the equations which govern the pressure distribution and force on the sphere in the lubrication limit.

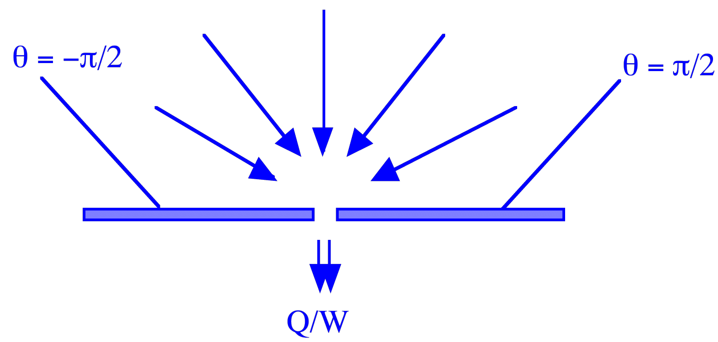
b. Using scaling analysis, determine the force necessary for the sphere to have a vertical velocity V for small h_0/a to within some unknown numerical value. Note that the appropriate length scale for r is -not- the radius of the sphere!

The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 2). (15 points) Two Dimensional Flows: Consider the two-dimensional geometry depicted below. A very viscous fluid is draining out of the bottom of a tank through a narrow slit at a rate Q/W per unit extension into the paper.



a. Set up the problem as completely as possible, with all boundary conditions clearly determined.

b. Using the boundary conditions as a guide, determine the form of the streamfunction, and solve for it. Make maximum use of symmetry!

c. Using the streamfunction calculated in part b., determine the shear stress distribution along the bottom as a function of r. (Hint: the answer may surprise you...)

Remember that in the cylindrical geometry we have:

$$u_\theta = -\frac{\partial \psi}{\partial r} \quad ; \quad u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

and

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}$$

where, in general,

$$\psi \sim r^\lambda f_\lambda(\theta)$$

$$f_\lambda(\theta) = A\sin(\lambda\theta) + B\cos(\lambda\theta) + C\sin((2-\lambda)\theta) + D\cos((2-\lambda)\theta)$$

with special cases (due to repeated roots):

$$f_0(\theta) = f_2(\theta) = A + B\theta + C\sin(2\theta) + D\cos(2\theta)$$

$$f_1(\theta) = A\sin(\theta) + B\cos(\theta) + C\theta\sin(\theta) + D\theta\cos(\theta)$$

and

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

Problem 3). (15 points). Dimensional Analysis: In what is arguably the largest environmental remediation project in history, the Hanford waste treatment facility is being constructed to separate and vitrify the liquid radioactive waste left over from the fabrication of atomic weapons. A key component of the process is the mixing vessels to homogenize the liquid waste. The mixing is done via pulse jet mixers, which are basically downward pointing bottles where liquid is first sucked in and then squirted out in an oscillatory manner. Your task is to develop the scaling laws necessary for designing a scale model test system for studying and evaluating the full scale system.

Full Scale System Parameters:
Tank Volume = 500,000 liters
Fluid Viscosity = 30 centipoise
Fluid Density = 1.4 g/cm^3

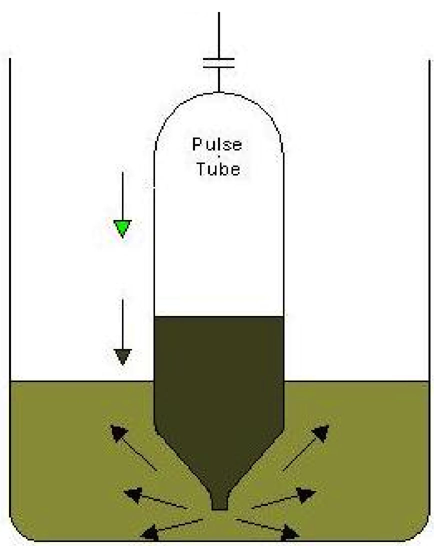
You want to model this system using water as the working fluid.

a. What should the volume of the model tank be?

b. If the operating pressure of the full scale tank pulse jets is ΔP_1, what should the pressure of the jets in the model system be?

c. If the period of oscillation of the full scale pulse jets is T_1, what should the period of the model scale pulse jets be?

Hint: Don't forget that the free surface in the pulse jets matters, and use strict dynamic similarity!



Problem 4). (15 points) Short Answer / Multiple Choice:

(2 pts each) Briefly identify the physical mechanism (or equation name) described by each of the following terms **and a problem where it would play a role**:

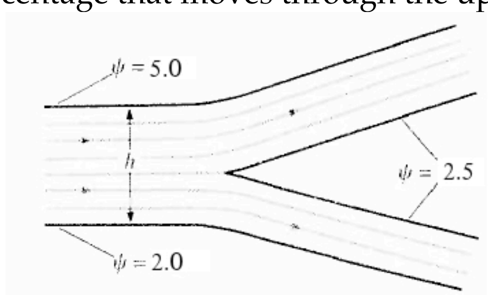
1. $\frac{\partial \sigma_{ij}}{\partial x_j} = 0$

2. $E^4 \psi = 0$

3. $\varepsilon_{ijk} \frac{\partial u_k}{\partial x_j} = 0$

4. It is proposed to model the flow patterns in a microfluidic chip of characteristic channel depth of 25μm and velocity of 1cm/s with a macroscopic scale model of channel depth 2mm. If the working fluid of the chip is water, and that of the model is a mixture of glycerin and water with kinematic viscosity 0.2cm^2/s, what should the velocity of the fluid in the model be?

5. Computer fluid simulations were carried out in a 2-D asymmetric channel bifurcation as presented below. The simulation reveals the stream function values at the channel walls. What is the flow percentage that moves through the upper branch of the channel?



6. Prove that inertial effects always increase the drag on an object relative to that caused by viscous effects alone. Be brief!

7. In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. Recent results from the Mars missions have shown that it once had seas much like Earth's. Given that the gravity on Mars is only 0.38 that of Earth, what would the maximum velocity of a 25m ship have been on Mars?

8. (1pt) My big orange goldfish, alas, passed away, and the little one left in the office is only about 1.5cm wide. Unless it is hungry, it also usually moves with a velocity of around 0.5cm/s. Is its motion dominated by inertial or viscous effects? Be quantitative!