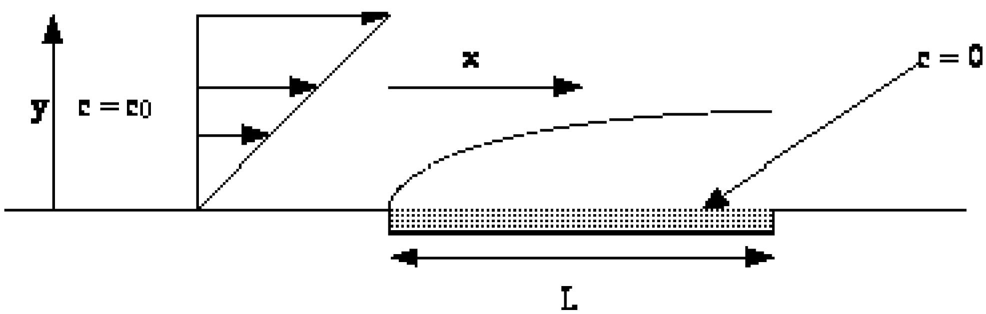
CHEG 355 Transport Phenomena I
Final Exam

December 17, 2004

Closed Books and Notes

Problem 1. (30 points) Scaling analysis of boundary layer flows. A popular method for measuring instantaneous wall shear stresses in turbulent flows is the use of an electrochemical probe. The fluid is doped with some reactant at concentration c_0 which decomposes electrochemically at an electrode flush with the wall. The reaction is essentially instantaneous, so the concentration of reactant at the electrode is zero, and the reaction rate is determined by the rate with which the stuff diffuses to the wall. By measuring the current, we can determine the reaction rate and hence the integrated mass flux to the electrode. The mass flux gets related to the shear stress because diffusive transport of mass is so slow relative to momentum transport that the mass transfer boundary layer essentially samples the linear shear flow right at the wall. We thus have the following problem:



$$\underset{\sim}{u} \cdot \underset{\sim}{\nabla} c = D \nabla^2 c \quad ; \quad \underset{\sim}{u} \sim \underset{\sim}{\hat{e}}_x \frac{\tau_w}{\mu} y$$

$$N \Big|_{y=0} = -D \frac{\partial c}{\partial y} \Big|_{y=0} \quad ; \quad I = W \int_0^L N \Big|_{y=0} dx$$

a. Using scaling analysis, determine how the current per unit width I/W scales with the parameters of the problem (e.g., τ_w, D (the diffusion coefficient), μ, c_0, and L).

b. Estimate the time resolution of such a probe. Choose L = 1mm, D = 10^{-5} cm^2/s, τ_w = 1000 dynes/cm^2, and the properties of water.

c. The current is only related to the shear stress if the boundary layer approximation holds. In general, we would like to make the probe as short as possible to get the best time resolution possible. What is the shortest length the probe can be?

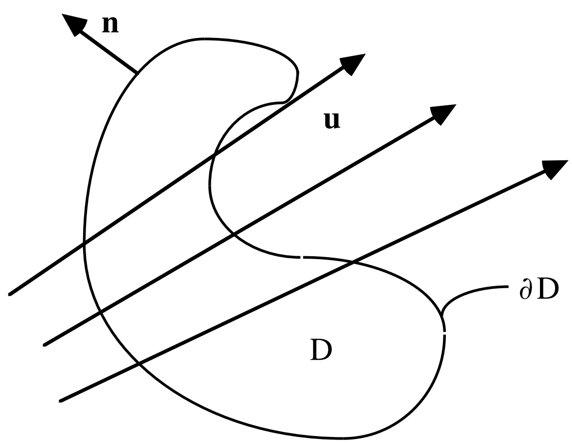
d. The concentration profile in the boundary layer is governed by a similarity solution. Derive the similarity rule, similarity variable, transformed differential equation, and boundary conditions in canonical form for this boundary layer problem.

Problem 2. (20 points) Derivation of transport equations. Consider an arbitrary fluid element D with surface ∂D and unit normal $\underline{\mathbf{n}}$. If the thermal energy per unit volume is given by $\rho C_p T$, and the diffusive heat flux $\underline{\mathbf{q}}$ by:

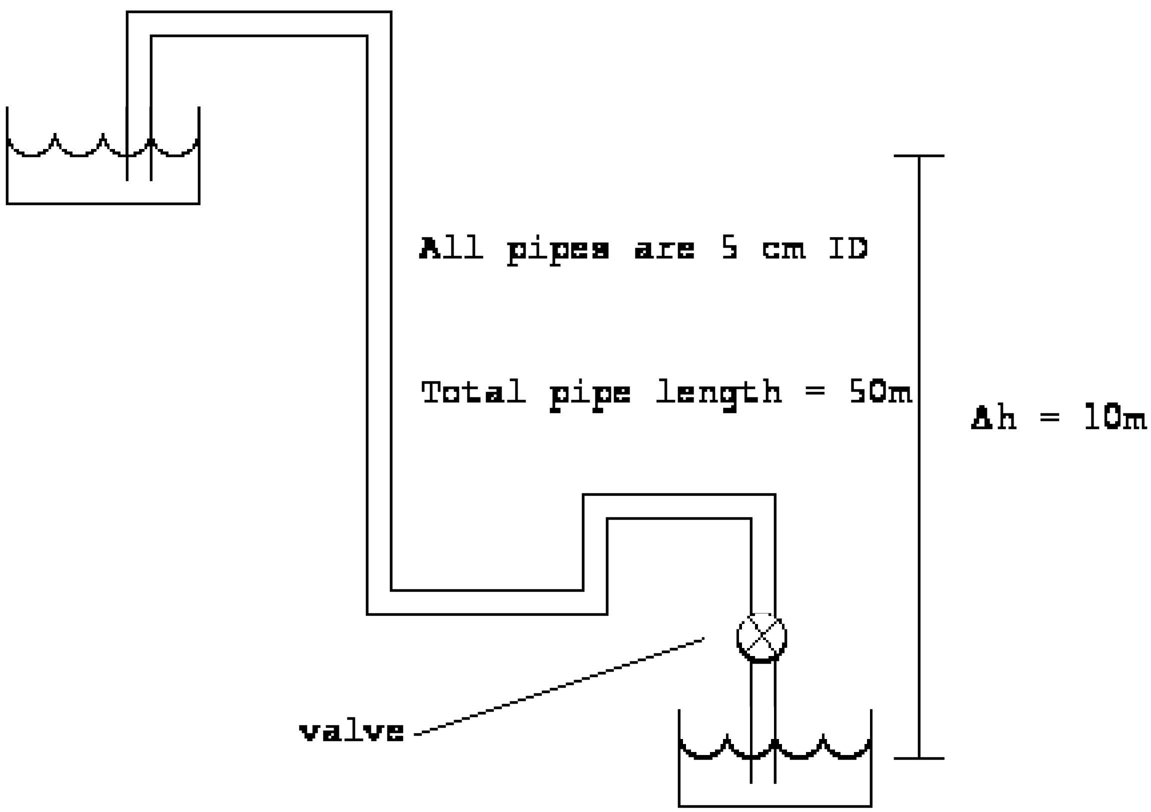
$$\underset{\sim}{q} = -k \underset{\sim}{\nabla} T$$

a. Write down an integral balance for the conservation of thermal energy in this system for an incompressible fluid taking into account accumulation, convection, and diffusion (no sources).

b. Derive the corresponding convective diffusion equation for constant physical properties.



Problem 3. (20 points) Pipe flow. Consider the piping network depicted below. Water drains from the upper reservoir to the lower reservoir through the 5cm ID siphon as shown. The rate at which the water flows through the system is controlled by the valve shown. What "K" value of the valve is required to maintain a flow of 5 liters/s? Friction factors for the pipes and fittings are given below.



You may find the following expressions useful:

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \, f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re} \sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first five questions refer to the pump curve on the last page:

1. It is desired to pump 100 liters/sec from a pond to an elevation of 35 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP200 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done <u>by</u> the pump <u>on</u> the fluid?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm $\approx$ 10.3 m water)

6. Write down the Navier-Stokes equations in index notation.

7. The turbulent friction velocity is a characteristic velocity created from a shear stress and fluid properties. What is it in terms of these parameters?

8. The turbulent viscous length scale is a characteristic length created from a shear stress and fluid properties. What is it in terms of these parameters?

9. What is the approximate thickness of the viscous sublayer in plus units?

10. For a shear stress of 100 dynes/cm^2 in the flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

11. The properties that give Super Soaker water guns their capacity and range are:
A. The compressibility of both water and air
B. The compressibility of water and incompressibility of air
C. The incompressibility of water and the compressibility of air
D. The incompressibility of both water and air

12. The purpose of the ridges on Frisbees is to:
A. Induce Poiseuille Flow
B. Delay boundary layer separation
C. Provide gyroscopic stability
D. Both B and C

13. Experimentally, how can you most accurately calculate the shear stress on a plate in boundary layer flow at high Reynolds numbers from the velocity profile?

14. Give a physical description of the Reynolds stress (e.g., where does it come from?).

15. Sketch and briefly explain the principle behind a venturi flow meter.

SYKES PUMPS

CURVE: CP2000108 ISSUE 3
DATE OF ISSUE 9 AUGUST 2002

PUMP : CP-200

SUCTION	DISCHARGE	MAX. SPHERE	IMPELLER	IMPELLER	IMPELLER & WEAR PLATES
200mm	200mm	80mm	3 VANE	⌀295mm	316 S/S