CBE 30355 Transport Phenomena I
Final Exam

December 18, 2008

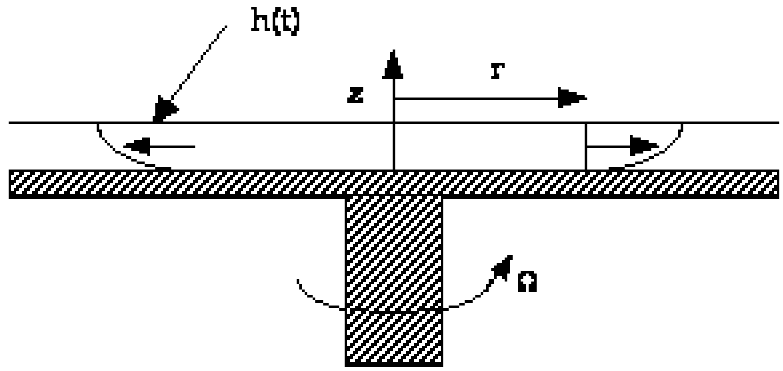
Closed Books and Notes

Problem 1. (20 points) Scaling/Lubrication: A common way to produce a thin film on a surface is to spin coat it - take a disk of radius R, deposit some liquid of kinematic viscosity ν on the surface, and then rotate it with some angular velocity Ω. Centrifugal forces cause the fluid layer to thin out over time, leaving a final thickness $h_f \ll R$ at the end of the process. Here we analyze this procedure.

a). For *thin* fluid layers the rotational velocity is just $u_\theta = \Omega r$ over the entire film (e.g., it moves with the rotational velocity of the disk). Using this, determine the differential equations and boundary conditions which govern the radial and vertical velocity distribution in the film, and the equation for the change in film thickness over time. (Hint: How does the change in thickness with time relate to u_z at $z = h(t)$?) It is appropriate to use h_f, the final thickness, as the vertical length scale.

b). Using scaling analysis, determine how long the spin coating process takes as a function of the parameters of the problem to within the usual unknown O(1) constant.

c). Explicitly solve for the velocity distribution and the height as a function of time to get the O(1) constant. You may take the initial height to be H where $H/h_f \gg 1$.

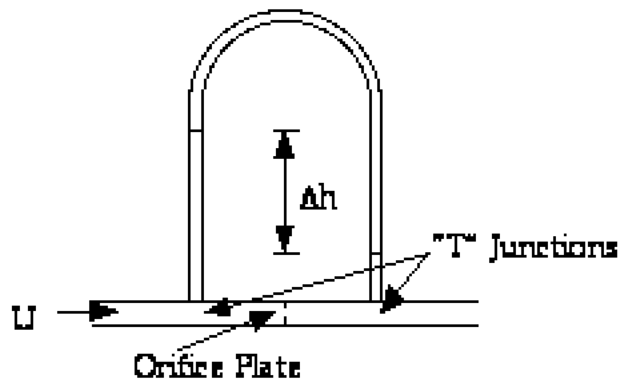


You may find the following equations helpful:

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 2. (20 points) Orifice meters. Consider the simple orifice meter depicted below. An inverted "U" tube, partially filled with air, is connected by a couple of "T" junctions separated by a short length of pipe. In between the "T"s is an orifice plate which contricts the flow and induces a pressure differential which is related to the water flow rate. The orifice reduces the area of the flow by a factor of 2 for this particular model (e.g., $\beta = 0.5$) and introduces a "K" value as given in the formula below.



a. Neglecting the pipe friction factors (e.g., take the length of pipe between the "T"s to be short), determine the relationship between the measured Δh in the U-tube and the flow rate. The pipe diameter is D, and the fluid density is ρ. Take "T" resistances to be distributed symmetrically to upstream and downstream sides of the junctions.

b. If Δh is measured to be 20 cm, calculate the velocity in the pipe and the total flow rate. Use the properties of water, and take the pipe diameter to be 2 cm.

c. A colleague argues that you need to also include the frictional losses in the length of pipe between the "T" junctions. While this is in principle correct, it may not be a significant correction. Determine if this is the case if the length of pipe between the "T"s is 20 cm.

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
T (through side)	1.5
T (straight through)	0.4
Sharp Orifice	$2.7\,(1-\beta)\,(1-\beta^2)\,\frac{1}{\beta^2}$

Problem 3. (20 points) Uni-directional Startup Flows. Consider the channel depicted below. For all times $t < 0$ the lower plate and fluid are at rest. At $t = 0$ the lower plate is accelerated such that the shear stress at the wall is a constant τ_w independent of time. The flow may be regarded to be unidirectional in the x-direction.

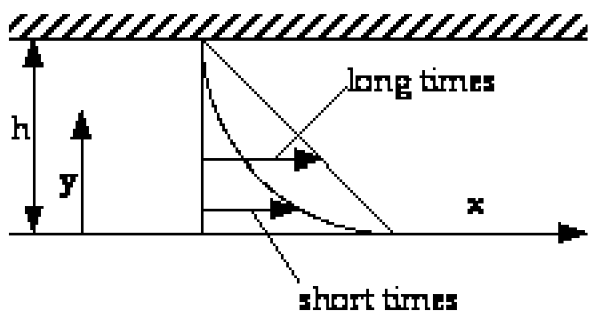
a. Determine the differential equation and boundary conditions governing this problem.

b. At very long times the flow reaches steady-state. Using t_c as this time scale and h as the length scale, how large does t_c have to be for the flow to become steady? (Hint: use scaling analysis and figure out what dimensionless group has to be large or small for the appropriate term to be thrown away). What is the velocity profile in this limit?

c. For very short times a boundary layer develops near the accelerating plate. Using t_c as this time scale, determine the characteristic thickness of this boundary layer (e.g., the new length scale) and the characteristic velocity of the lower plate.

d. Using the technique of coordinate stretching, show that this short-time limit admits a self-similar solution and obtain the similarity rule and variable in canonical form. Determine the velocity of the lower plate as a function of time to within an unknown O(1) multiplicative constant (e.g., the solution of the similarity problem).

e. Approximately when will this solution no longer be valid?



Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first seven questions refer to the pump curve on the last page:

1. It is desired to pump 100 liters/sec from a pond to an elevation of 60 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH150 recommended for the job?

2. What is the RPM required to do the job?

3. What is the mechanical work done <u>by</u> the pump <u>on</u> the fluid per unit time?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

7. It is proposed to use a 10cm diameter pipe for this system. If we include just the losses due to the initial contraction (K=0.4) and acceleration of the fluid, how does the answer to question 5 change?

8. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

9. Briefly describe one method for preventing stall on an aircraft wing.

10. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

11. The turbulent friction velocity is a characteristic velocity created from a shear stress and fluid properties. What is it in terms of these parameters?

12. For a shear stress of 16 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

13. Experimentally, how can you most accurately calculate the shear stress on a plate in boundary layer flow at high Reynolds numbers from the velocity profile?

14. Sketch and briefly explain the principle behind a venturi flow meter.

15. Provide two interpretations of $\rho \underset{\sim}{u}$

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DATE OF ISSUE: 4 JUNE 2002

PUMP : HH150

SUCTION	DISCHARGE	MAX. SPHERE	IMPELLER	IMPELLER	IMPELLER & WEAR PLATES
200mm (8")	150mm (6")	38mm (1.5")	OPEN	ø398mm (15.7")	316 S/S