CBE 30355 Transport Phenomena I
Final Exam

December 18, 2009

Closed Books and Notes

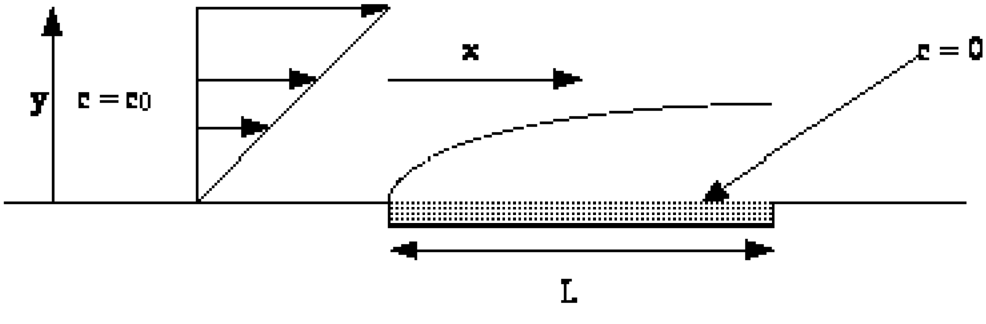
Problem 1). (20 points) Transient Boundary Layer Scaling. You are asked to design an electrochemical probe capable of measuring both concentration and wall shear stress next to a wall. The idea is that fluid flows along the surface of the wall containing some concentration c_0, and then at the surface of the probe we electrochemically reduce the concentration to zero over the whole length L. Initially, you develop a transient boundary layer independent of x, and then at long times you develop a steady, x-dependent boundary layer independent of t. The total reaction rate per unit width (extension into the paper) Q/W can thus be used to determine both c_0 and τ_w by examining these two limits.

a). Scale the equations and boundary conditions at short time scales t_c to determine how the transient boundary layer thickness δ_t and Q/W depend on the parameters of the problem.

b). Scale the equations and boundary conditions at long times to determine how the steady boundary layer thickness δ_s and Q/W depend on the parameters of the problem in this limit.

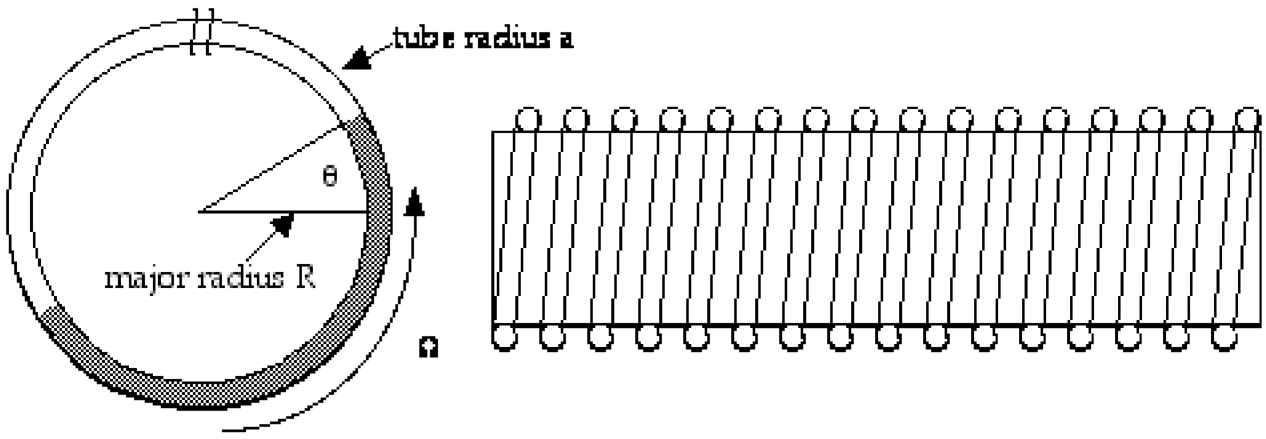
c). The transition between the short time and long time behavior occurs for some t_c where the boundary layer thicknesses in a and b are of the same order. What's this t_c?

d). If our electrochemical measurement system requires a time of 0.01 seconds to determine a decent measure of the reaction rate, the diffusivity is around 10^{-6} cm^2/s, and the wall shear rate is 1000 s^{-1}, what is the minimum length L of the probe? What would be the characteristic boundary layer thickness under these conditions?



$$\frac{\partial c}{\partial t} + \frac{\tau_w}{\mu} y \frac{\partial c}{\partial x} = D\left[\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right] ; \qquad \frac{Q}{W} = \int_0^L D \frac{\partial c}{\partial y}\bigg|_{y=0} dx$$

Problem 2. (20 points) Coil pumps again. You are designing a coil pump to supply water to an irrigation system. The water needs to be pumped up the bank 3 meters (e.g., that is the required head output of the pump), and you need a water flow rate of 1 liter/s. You have available a rotating drum with a major **radius** of 20 cm and tubing (to coil around the drum) of 2 cm **radius**.



a. Neglecting losses, what is the minimum number $\mathbf{N_0}$ of coils that are necessary to get the fluid up to the required height? Assume that the back pressure is divided uniformly among the coils, and that each of the coils is half-filled with water.

b. What is the angular velocity (e.g., radians/s) necessary to achieve the desired flow rate? What is the Reynolds number?

c. An empirical study suggests that the head losses in **each** coil are equivalent to a "K" factor of two long-radius 90°elbows (0.4 per elbow) plus the friction factor due to the pipe itself. Ignoring entrance and exit effects into the pump, determine the corrected minimum number of coils $\mathbf{N_{min}}$ (a bigger number than that in part a!).

d. Will entrance and exit effects have a significant effect on N_{min}? Be quantitative!

e. In general it is a good idea to have more coils (**N**) than the minimum number! What is the mechanical efficiency of the pump as a function of the number of coils? You should be able to express this in terms of N, N_0, and N_{min} alone! (Hint: this is most easily calculated by calculating the mechanical work out and the total hydrodynamic losses in the system).

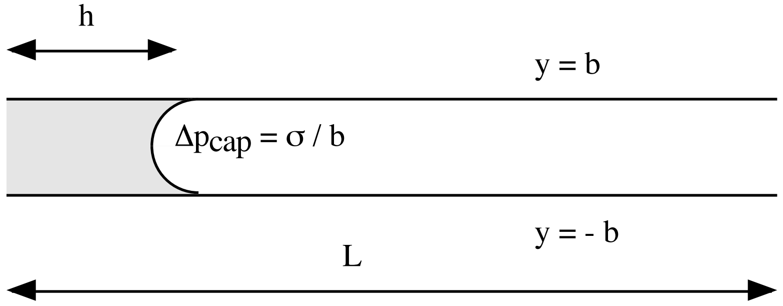
$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;\; ; \; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;\; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \;\; ; \; \mathrm{Re} > 3000$$

Problem 3. (20 pts) Analysis of blood chemistry is now being done by putting a tiny drop of blood onto the entrance of a microchannel, and the blood is drawn into the channel via capillary action. This is the same effect that allows fuel to be drawn up the wick of a lamp, for example. In this problem we examine the length of time necessary for the channel to fill.

Consider the channel of depth 2 b and length L given below. The curvature of the fluid interface causes the pressure behind the meniscus to be less than atmospheric pressure by an amount $\Delta p_{cap} = \sigma/b$. Because the channel dimensions are very small, inertial effects are negligible and the force due to the capillary pressure is balanced by viscous effects only.



a. Using this information, determine the equations governing the velocity distribution in the channel, and the length h of the column of fluid in the channel as a function of time. Make any approximations or simplifications necessary to get a reasonable problem. (Hint: think of plane-Poiseuille flow, flow rates, uni-directional flow, and mass balances)

b. Using scaling analysis, render the problem dimensionless and determine the dependence of the fill time on all the parameters of the problem.

c. Solve for the actual value of the fill time with the initial condition h(0) = 0 (e.g., the channel is initially empty).

(As a side note, the analysis you do above works very well provided the channel depth is greater than around 50μm. For microchannels thinner than this value, some fascinating migration effects of the red blood cells become significant, and you have to do a much more sophisticated analysis of the problem!)

Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first seven questions refer to the pump curve on the last page:

1. It is desired to pump 125 liters/sec from a pond to an elevation of 65 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH150 recommended for the job?

2. What is the RPM required to do the job?

3. What is the useful mechanical work done <u>by</u> the pump <u>on</u> the fluid per unit time?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

7. It is proposed to use a 10cm diameter pipe for this system. If we include just the losses due to the initial contraction (K=0.4) and acceleration of the fluid, how does the answer to question 5 change?

8. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

9. Moffat Eddies: What are they, and why are they important?

10. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined in terms of fluctuation velocities?).

11. For a shear stress of 25 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

12. Sketch and briefly explain the principle behind an orifice flow meter.

13. Write down Bernoulli's equation (include the gravity term!).

14. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

15. In our class demonstration, when we allowed a rod to settle in a viscous liquid (Re = 0), we observed that it didn't rotate. If inertial effects are important, it does. Why is the behavior different in this case?