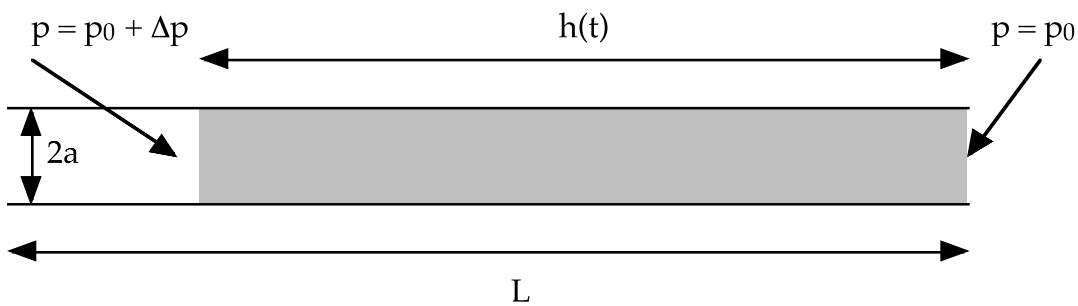
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 pts) Scaling/Unidirectional flows: Consider a horizontal straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below. The liquid is initially at rest. At time t=0 we start to blow the liquid out of the straw by applying a constant pressure **differential** Δp. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw in both the high and low Re limits - e.g., how long does it take for h to reach zero.



a. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: **unsteady**, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

c. Solve the problem to obtain the dimensionless non-linear second order ODE which governs the evolution of the liquid slug length in this limit, together with initial conditions. This equation is trivial to solve numerically using matlab, of course, (the dimensionless blowout time comes out to be (π/2)^.5) but don't do it here!

d. Scale the equations for LOW Reynolds numbers, and determine the new (unknown) characteristic blowout time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

The equations of motion in cylindrical coordinates are given below (you don't need all of these for this problem!):

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

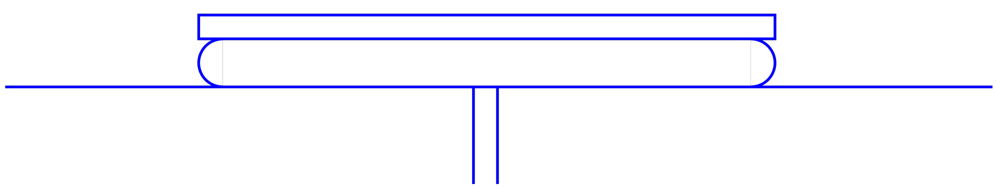
$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho\, g_z$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho\, g_r$$

Problem 2). (20 points) **Lubrication**: Consider a disk of radius R located a distance H above a plane as depicted below. Fluid of viscosity μ is pumped into the center of this geometry through a very small tube at a rate Q (volume/time), and flows outward radially. In contrast to the high Re problem demonstrated in class, at low Re the force on the disk is upward (e.g., the force is away from the plane). Here we analyze this problem in the lubrication limit $H/R << 1$.



a. Using a mass balance, determine the characteristic magnitude of the radial velocity and how it depends on r.

b. Write down the appropriate form of the equations of motion in the lubrication limit, keeping only the non-zero terms, and write down an integral relationship for the force on the plate.

c. Via scaling analysis of the equations of motion and a force balance on the plate, determine how the force scales with the parameters of the problem.

d. Solve the problem to obtain an explicit value for the pressure distribution and force.

Problem 3). (10 points) Dimensional Analysis: Some of your classmates demonstrated the phenomenon of hydroplaning. Here we examine where that could occur. You are traveling in a car which has (fairly bald) tires inflated to a pressure of 30psig. It hits a puddle of water on the road that is deeper than the remaining tread. Estimate the velocity at which you would expect it to hydroplane (e.g., when would all the tires lose frictional surface contact with the road). Don't just guess, show your reasoning!

Problem 4). (10 points) Dimensional Analysis: You are responsible for determining the size of the power plant and fuel consumption for a fleet of freighters with similar design (hull shape), but different sizes. All of them are traveling as fast as they can practically go on the same route. How do the transit time, size of the power plant, and total fuel cost vary with the freighter volume?

Problem 5). (20 points) Short Answer / Multiple Choice:

a. (1 pt each) Briefly identify the following:

1. Stokes Paradox
2. D'Alembert's Paradox
3. Osmotic Pressure
4. Ideal Flow
5. Electroosmosis
6. Cold Gas Approximation
7. Brownian Motion
8. Stokeslet
9. Richardson Number
10. Poiseuille's Law

b. (10 points) Write down the Navier-Stokes equations in index notation, and describe the physical mechanism giving rise to each of the five terms.