CBE 30355 TRANSPORT PHENOMENA I

First Hour Exam
10/3/17

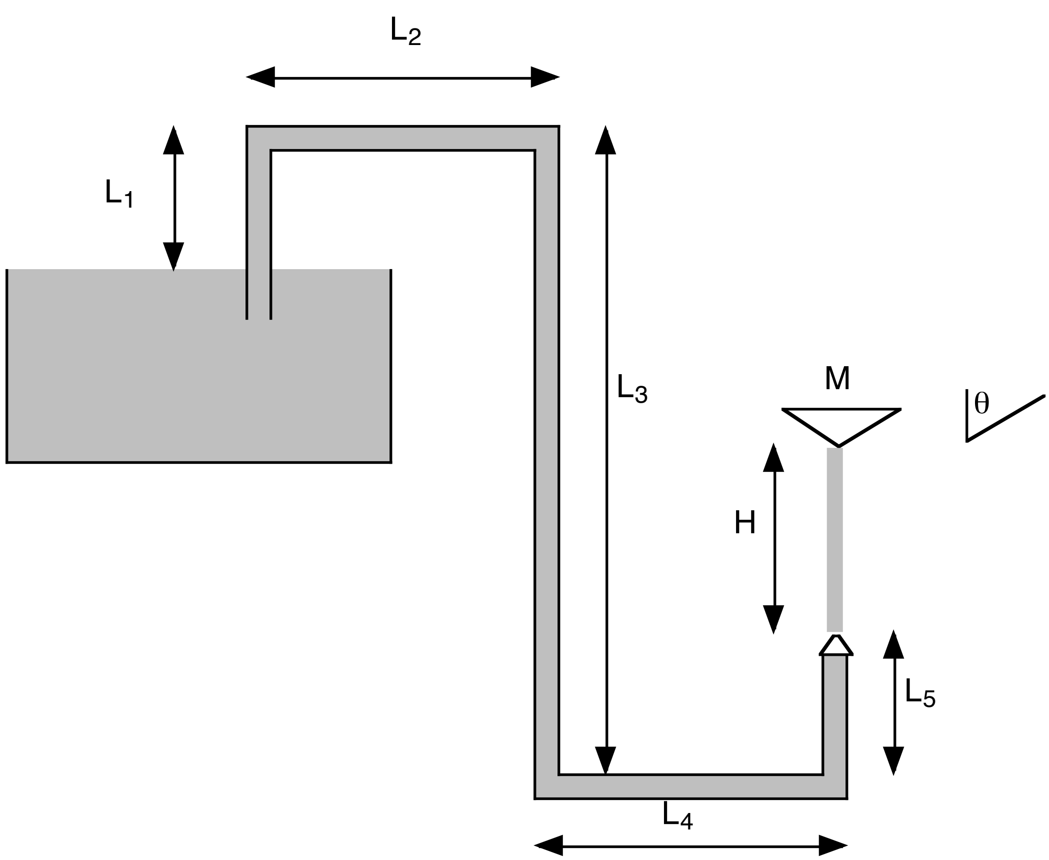
This test is closed books and closed notes

Problem 1). (20 pts) Integral Momentum Balances: Consider the gravity driven fountain depicted below. Water from a reservoir travels through a pipe to feed a fountain with a nozzle of area A. The water from the jet then supports a mass M, where the "deflector" is a cone with internal angle 2θ. The objective here is to analyze this fountain and figure out how high the mass floats above the fountain nozzle.

a. Neglecting all losses (actually not a bad approximation if the nozzle area is a lot smaller than the pipe diameter) develop an expression for the flow rate of the fluid through the jet.

b. If M is really small, what is the maximum height it can reach at steady-state?

c. Using an integral momentum balance, develop an expression for the height of the mass M as a function of the cone angle and the other parameters in the problem.



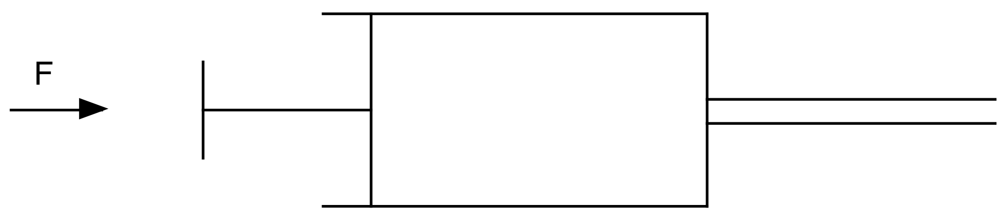
Problem 2). (20 points) Poiseuille Flow: A syringe with radius R_1 and length L_1 is being emptied through a needle with radius $R_2 << R_1$ and length L_2 in time T as depicted below.

a. Using a mass balance determine the ratio of the average velocity in the needle to that in the syringe.

b. Assuming all losses to be in the needle, and assuming fully developed laminar flow, derive and expression for the force on the plunger necessary to empty the syringe in time T.

c. If $R_1 = 0.5$cm, $R_2 = 0.05$cm, $L_1 = 4$cm, $L_2 = 2$cm, T = 4s, and the working fluid has the same viscosity as water, determine the numerical value of this force.

d. Quantitatively comment on the validity of the assumption of fully developed laminar flow in the needle.



The equations of motion in cylindrical coordinates are given by:

$$r:\quad \rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\phi^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \phi^2}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\phi}{\partial \phi}\right]+\rho g_r$$

$$\phi:\quad \rho\left(\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_\phi}{\partial \phi}+u_z\frac{\partial u_\phi}{\partial z}+\frac{u_r u_\phi}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \phi}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\phi}{\partial \phi^2}+\frac{\partial^2 u_\phi}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \phi}-\frac{u_\phi}{r^2}\right]+\rho g_\phi$$

$$z:\quad \rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\phi}{r}\frac{\partial u_z}{\partial \phi}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \phi^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z.$$

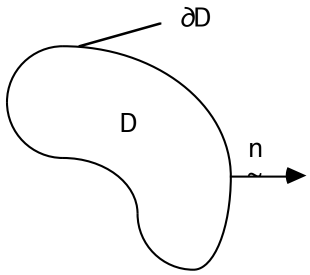
$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial(\rho u_\phi)}{\partial \phi}+\frac{\partial(\rho u_z)}{\partial z}=0.$$

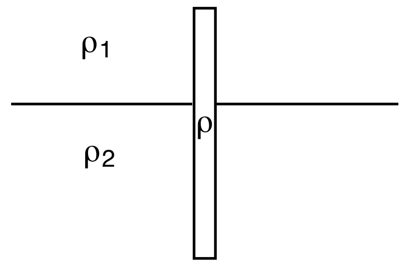
(note that in this version ϕ was used for the angular coordinate where we used θ in class)

Problem 3. (20 pts) Hydrostatics:

a. From an integral force balance over the arbitrary object depicted below, derive Archimedes Law for a fluid at rest.

b. A cylinder of radius R and length L is immersed at the interface of two fluids as depicted below. If the density of the object is ρ, and the densities of the two fluids are ρ_1 and ρ_2 such that $\rho_1 < \rho < \rho_2$, use Archimedes Law to determine the fraction of the object immersed in the second fluid.





Problem 4). (20 pts total) Transport Glossary / Index Notation / Short Answer

a. (6pts) Briefly identify the physical mechanism described by each of the following terms:

1. $\rho \frac{\partial u_x}{\partial t}$

2. $\tau_{ij} \quad vs. \quad \sigma_{ij}$

3. $\rho \frac{u_\theta^2}{r}$

b. (2 pts) What is the ratio of the centerline velocity to the average velocity for a) laminar flow in a tube, and b) laminar flow in a channel?

c. (2pts) Give and example of 1). a pseudo vector and 2). a second order physical tensor discussed in class.

d. (2pts) What is the most general relationship for the velocity of a body of revolution falling under some force F_j in terms of its orientation vector p_j at zero Re? Use index notation.

e. (2 pts) Does Poiseuille's Law govern pressure drop / flow rate relationships for your typical house plumbing system? Briefly (quantitatively) justify your answer.

f. (2 pts) Match up the kinematic viscosities of the following materials:

1. Glycerin	A. 0.118 cSt
2. Air	B. 1.0 cSt
3. Water	C. 17.0 cSt
4. Mercury	D. 650 cSt

g. (2 pts) What is the continuum hypothesis, and where does it break down?

h. (2pts) . A glass sphere of radius 5mm with thermal diffusivity $\alpha = 10^{-2}$ cm^2/s is dropped into hot oil. About how long will it take for the temperature at the center of the sphere to equilibrate?