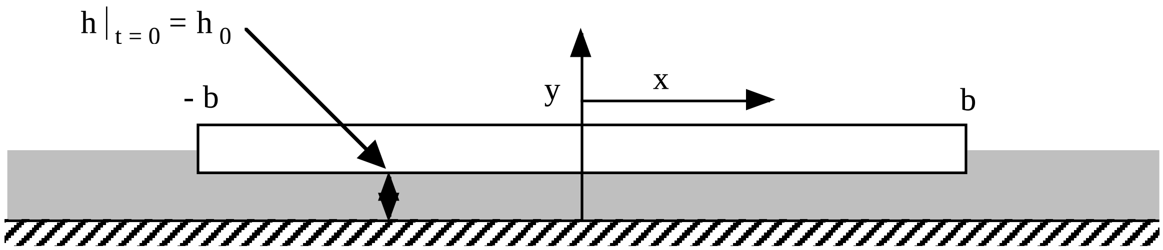
CHEG 355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Lubrication Theory: In class we demonstrated the detachment of a disk from a plane. In this problem we solve the corresponding 2-D problem of the detachment of a long strip of width 2 b from a plane. If we apply an upward force per unit length (extension into the paper - the direction that doesn't matter) of F / L, what is the time for the strip to separate from the plane given an initial separation distance h_0?



a. Write down the continuity equation and the x-momentum equation, render the equations dimensionless, and determine which terms are significant in the lubrication limit.

b. Using inspectional analysis, derive the equation governing the separation as a function of time to within some unknown integral (e.g., don't solve the lubrication problem yet - just do the scaling).

c. Solve the lubrication equations to explicitly obtain the pressure distribution and separation time. (This part takes alot of time, so don't do it until you've worked on the other questions!)

Problem 2). (20 points) Derivation of Transport Equations: There are many parallels between momentum, mass and energy transport since all three are derived from similar conservation laws. In this problem we derive a microscopic balance describing the concentration distribution $c(\underset{\sim}{x},t)$ of a very dilute suspension of small particles suspended in an incompressible fluid undergoing unsteady flow. (Note: $c(\underset{\sim}{x},t)$ is the local volume fraction of particles in the fluid - volume of particles / volume of fluid, and hence is dimensionless.)

a. The flux of particles is the sum of that due to convection, that due to settling $\underset{\sim}{q}_s$ (a vector), and that due to diffusion $\underset{\sim}{q}_D$ (also a vector). These fluxes have units of volume per area per time (e.g., velocity). With this in mind, write down an integral balance for the conservation of particles for an arbitrary control volume D. Using the divergence theorem, convert all surface integrals to volume integrals, and so obtain a microscopic equation valid at any point in the flow field.

b. For small particles of radius a, the flux due to settling is given by:

$$\underset{\sim}{q}_s = \frac{2}{9}\frac{\Delta\rho\, a^2}{\mu}\,\underset{\sim}{g}\, c$$

and the diffusive flux by:

$$\underset{\sim}{q}_D = -\frac{kT}{6\pi\mu a}\,\underset{\sim}{\nabla} c$$

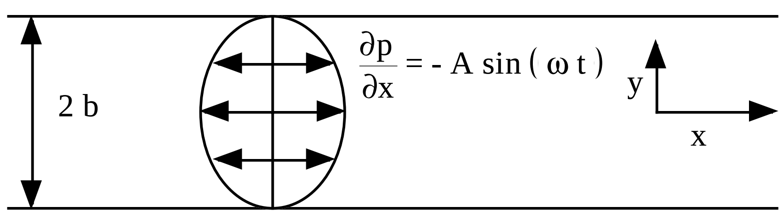
Using these results, obtain an equation in terms of derivatives of the concentration.

c. Using a reference velocity U, reference length L, and reference time L / U, render the equation dimensionless. What dimensionless parameters appear and what is their physical significance? This equation is the starting point for the study of many problems involving suspensions of particles for which Brownian motion is significant (it was Brownian motion that gave rise to the diffusion term given above).

Problem 3). (20 points) Inspectional Analysis: Consider the **unsteady** oscillatory flow in a channel of width 2 b depicted below. The fluid is incompressible, and the flow is unidirectional in the x-direction, with all that implies (hint: remember which of the inertial terms vanish!). We impose an oscillatory pressure gradient given by:

$$\frac{\partial p}{\partial x} = -A \sin(\omega t)$$

where A is the gradient amplitude and ω is the frequency of oscillation in time (the fluid sloshes back and forth in the x-direction).



a. Write down the momentum equation in the x-direction and show which terms are zero.

b. Render the equations dimensionless using $t^* = \omega t$ as the dimensionless time and U_C as an unknown velocity scale. Divide out by A to make the equations dimensionless.

c. The characteristic velocity U_C is determined by balancing the driving force in the problem (the pressure gradient) with either the inertial or viscous term. Recognizing this, determine this characteristic velocity for 1) high, and 2) low frequencies, and explicitly identify the single dimensionless group the problem depends on in either case (hint: it's a strange looking Reynolds number).

Problem 4). (10 points) Index Notation: Write down the continuity equation and Navier Stokes equations for an incompressible Newtonian fluid in index notation.

Problem 5) (10 points) Short Answer:

1). Which of the following operate in an inertia dominated regime? (give all that apply)

A. Space Shuttle
B. Motor Boat
C. Minnow
D. Paramecium
E. Cytological Streaming

2). Which (if any) of the following are discontinuous at a fluid-fluid interface?

A. Velocity
B. Shear stress
C. Heat flux
D. Mass flux
E. Shear rate

3). True or False: In CFD a computer numerically solves a differential equation by approximating it with a set of algebraic equations.

4). If we draw a line of dye or bubbles across a flow field and then watch it evolve, we get a:

A. Streakline
B. Pathline
C. Timeline
D. Streamline
E. Streamfunction

5). An ice skate slides on ice because:

A. Pressure concentration beneath the blade melts a thin layer of water
B. Frictional heating melts a thin layer of water
C. The coefficient of friction of frozen water is very low