CBE 30355 **<u>TRANSPORT PHENOMENA I</u>** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

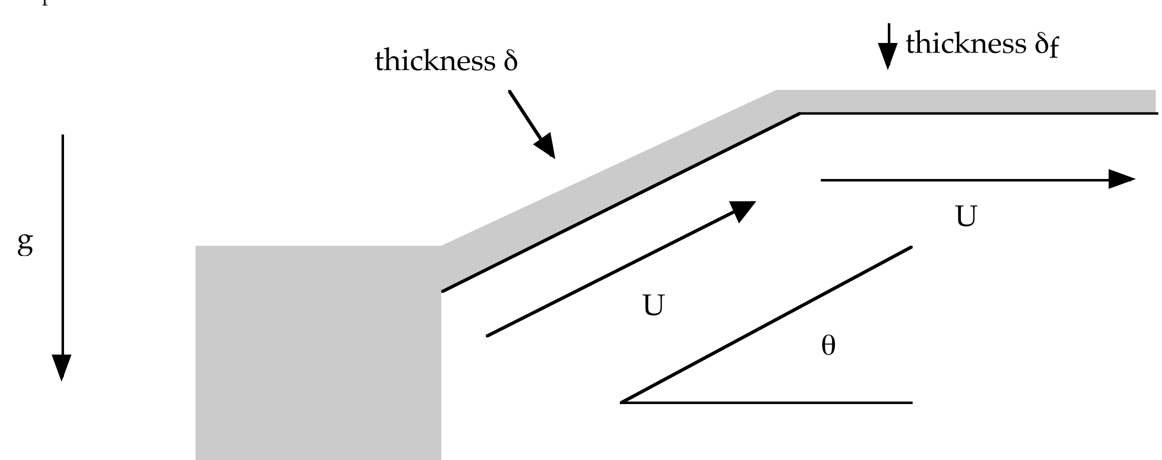
Note: Problems are not equally weighted!

Problem 1). (10 points) Dimensional Analysis: Dip coating with a viscous fluid. Dip coating (where an object is dipped in and then withdrawn from a vat of fluid, leaving a residue of fluid on the surface) is a commonly used coating technique.

a. For the case of strong surface tension Γ (the most commonly encountered case), the thickness of the film δ_f is a function of the vertical velocity U, the viscosity μ, the surface tension Γ (units of Force/Length), and the buoyant force ρg (inertia is small, so it doesn't depend on inertia (density) separately). Using dimensional analysis, determine the number of independent dimensionless groups involved in the problem, and show what group(s) the problem depends on. (Hint: This is fairly simple if you used the dimensional matrix to construct characteristic lengths (groups of parameters with units of length), and then the dimensionless groups are just ratios of these length scales.)

b. A recent paper (Snoeijer, et al., Phys Rev. Let., 2008) demonstrated that under some conditions thicker films can also occur in dip coating. In this case, the thickness no longer depends on Γ. Using this information, repeat the dimensional analysis of part (a) and determine this maximum thickness to within an O(1) constant.

Problem 2). (20 points) Continuous coating. A problem closely related to problem 1 is depicted below. A sheet is coated with a viscous liquid by being drawn through a bath and up an incline at some angle θ at some velocity U. The coated sheet then enters a horizontal area where the liquid solidifies (e.g., over this region the coating moves with the velocity of the belt). Your mission is to calculate the final thickness of the coating film δ_f in the horizontal area.



a. How does the final film thickness δ_f depend on the net fluid flow up the incline (per unit width "into the board") Q/W and the belt velocity U? (This is easy!)

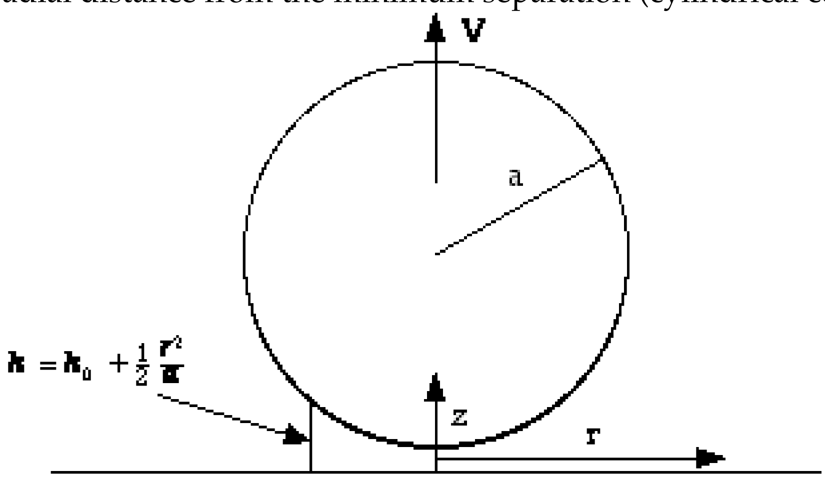
b. The thickness of the coating fluid on the incline δ will be different from the thickness δ_f in the horizontal area due to the different velocity profiles. Calculate how Q/W depends on δ and the parameters of the problem.

c. If surface tension is negligible, the thickness of the coating film on the incline will be that which results in the maximum Q/W. Using this information, determine this thickness and use it to calculate the final thickness δ_f as a function of the parameters of the problem.

Problem 3). (20 points) Lubrication: Detachment of a Sphere from a Plane: Consider the geometry depicted below. A sphere of radius a is separated from a plane by a distance h_0 such that $h_0/a << 1$. In the narrow gap limit, the gap width h is given approximately by:

$$h = h_0 + \frac{1}{2}\frac{r^2}{a}$$

where r is the radial distance from the minimum separation (cylindrical coordinates).



a. Determine the equations which govern the pressure distribution and force on the sphere in the lubrication limit.

b. Using scaling analysis, determine the force necessary for the sphere to have a vertical velocity V for small h_0/a to within some unknown numerical value. Note that the appropriate length scale for r is -not- the radius of the sphere!

The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 4). (10 pts) Dimensional Analysis: You are assigned the task of estimating the drag on a 100 meter long destroyer moving at 10 m/s (roughly 20 knots, or 20 nautical miles per hour). You choose to do this using a towing tank with a 1 meter long model.

a. The dimensionless drag depends on two key dimensionless parameters: what are they, and what is their approximate magnitude for the full-sized ship? (Hint: If you can't remember them, you can easily get them by appropriate scaling of the Navier-Stokes equations!)

b. Using the concept of approximate (rather than strict) dynamic similarity, determine the velocity at which the model should be towed to best approximate the flow pattern of the full-sized ship. What are the values of the important dimensionless parameters for the model and what is the ratio of the drag on the destroyer to that measured for the model?

Problem 5). (10 points) Short Answer / Multiple Choice:

Write out the term which corresponds to the following physical mechanisms:

1). Diffusion of x-momentum in the y-direction.

2). Convection of x-momentum in the y-direction.

3). The coriolis force, and which component of the NS equations (r, θ, or z) does it appear in?

4). A paramecium is a single celled animal around 200μm in size that tends to swim around in circles with a velocity of 1mm/s. Estimate its Reynolds number.

5). In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 10m boat of this type?